Measuring the inadequacy of IRR in PFI schemes using profitability index and AIRR

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Abstract. The internal rate of return (IRR) is widely used in Private Finance Initiative (PFI) schemes in the UK for measuring performance. However, it is well-known that the IRR may be a misleading indicator of economic profitability. Treasury Guidance (2004) recognises that the the IRR should not be used and net present value (NPV) should be calculated instead, unless the cash flow pattern is even. The distortion generated by the IRR can be quantified by the notion of *scheduling effect*, introduced in Cuthbert and Cuthbert (2012). We combine this notion with the notion of *average IRR* (AIRR), introduced in Magni (2010, 2013) and show that a positive scheduling effect arises if the AIRR, relative to a flat payment stream, exceeds the project's IRR. The scheduling component can be measured in two separate ways, in terms of specific AIRRs, one of which enables the scheduling component to be decomposed into relative capital and relative rate components. We also highlight the role of average capital, whose quotation in the market, in association with IRRs or AIRRs, would deepen the economic analysis of the project.

Keywords. PFI, AIRR, profitability index, scheduling effect, internal rate of return.

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1. Introduction

Economic profitability of projects is a major topic in production economics and engineering economy as well as corporate finance. In academia, the net present value (NPV) is often credited as the most theoretically reliable measure: Rubinstein (2003) defines present value a "great moment" in financial economics, while Brealey et al. (2011) include NPV in a list of the seven greatest ideas in modern finance. However, empirical surveys show that, in many circumstances, decision makers use other tools for evaluating investments in real-life applications: internal rate of return (IRR), residual income (e.g., EVA), return on investment, payback period, profitability index (Remer et al., 1993; Remer and Nieto, 1995a, 1995b; Slagmulder et al., 1995; Lefley, 1996; Graham and Harvey, 2001; Lindblom and Sjögren, 2009).[2] While a general consistency among some metrics such as benefit-cost ratio, profitability index, net future value, residual income is confirmed (see Peasnell, 1982; Magni 2009; Pasqual et al. 2013), the IRR suffers from several difficulties, which have been recognised long since (see Magni, 2013, for an updated compendium of IRR's flaws. See also Percoco and Borgonovo, 2012, on the different ranking of key drivers provided by NPV and IRR).

Such difficulties prevent the IRR from being a generally reliable alternative to NPV-based calculations for the measurement of an investment's economic profitability. Yet, the IRR is still a favoured metric in several contexts. In particular, in assessing projects undertaken using the Private Finance Initiative (PFI), the IRR is often employed as a convenient measure of different aspects of the project – like the cost of capital formation for the public sector or the return to investors. In its guidance notes, the UK Treasury endorses the use of NPV in PFI projects and warns against the IRR, admitting its use only if the relevant payment profiles are of a flat, annuity, type (Treasury, 2004); there is a rationale for this, because in the latter case knowledge of the IRR enables unravelling the cash flows and, if the initial capital investment is known as well, the NPV can be computed. When the scheduling of payment streams departs from the flat,

[2]Another widely used decision criterion is the Modified Internal Rate of Return (MIRR) (see Lin 1976, Biondi 2006, Kierulff 2008, Lefley 2015). However, unlike IRR, which aims at measuring the rate of return of the project, the MIRR measures the rate of return of a portfolio of the project *and* the reinvestment of interim cash flows, assuming a given external reinvestment rate: "it's a rate of return on a modified set of cash flows, not the project's actual cash flows" (Ross et al. 2011, p.250). As Brealey et al. (2011, p.141) put it: "The prospective return on another independent investment should never be allowed to influence the investment decision"

annuity, assumption, the use of IRR as an indicator may give a seriously misleading impression of actual PFI costs and returns.[3]

In a recent paper, Cuthbert and Cuthbert (2012) both gave empirical evidence showing that the assumption of flat payment profiles was commonly violated in PFI schemes and developed an approach, based on the definition of what the authors called interest and scheduling effects, which allowed measuring the distortion caused by the IRR. Separately, Magni (2010, 2013), developed a generalisation of the concept of IRR, denoted as *average internal rate of return* (AIRR).[4] This is based on the concept of the rate of return to an investor in a project, relative to any general stream of capital values which might be of particular relevance, rather than to the specific stream of outstanding capital values implied by a given IRR, which allows the distortion caused by the use of IRR to be measured.

What the current paper shows is how there are close links between these two approaches, in that the scheduling effect as defined by Cuthbert and Cuthbert can be simply expressed in terms of specific AIRRs. In fact, the paper develops two such expressions, in terms of different choices of AIRR. What this paper represents, therefore, is a significant consolidation of what might at first sight appear to be quite separate strands of investment appraisal theory.

More specifically, we start from a profitability index, and decompose it into a return component and a scheduling component, the latter describing the deviation of the net market value of the project with respect to the net market value the project would have if it were a flat, annuity type, investment. We find a significant relation between two versions of AIRR. In particular, the first version is tied to IRR-implied capital values and signals the existence of a scheduling effect if and only if the AIRR is greater or smaller than the IRR. The same result is reframed in terms of invested average capital. The second AIRR, called 'blended economic AIRR' (BEAIRR) is tied to market values and enables decomposing of the scheduling component into a relative capital component and relative return component. We also show that the scheduling component can also be described in terms of invested average capital. We then show a relation between the scheduling effect and a partial ordering on transaction vectors with the same IRR: if a scheduling effect arises, then the later the payments are scheduled the greater the AIRR. We also apply the results to three real-life PFI examples taken from hospital projects in Scotland and the North of England.

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[3] Furthermore, even in the favourable case of flat payments, ranking schemes with the IRR is inconsistent with the NPV ranking, unless further conditions are met (see Section 2).

[4] Most recently, Magni (2015) presented a modification of the AIRR, which can also be used for project evaluation.

Note that the approach presented in this paper, while it has been illustrated in relation to PFI schemes, has much broader applicability. Similar problems occur widely in relation to general investment schemes: and the use of the notion of scheduling effect, combined with the AIRR approach, enables a sophisticated analysis of the economic profitability of any investment scheme.

The structure of the paper is as follows. In section 2 we introduce the problem of IRR in relation with PFI schemes and summarise Cuthbert and Cuthbert's (2012) results; in particular, we decompose the profitability index into interest component and scheduling component. In section 3 we describe the AIRR approach and introduce the new notion of blended AIRR (BEAIRR). Section 4 shows that the scheduling effect can be captured by the comparison of AIRR and IRR as well as by the average invested capital which is directly drawn from the AIRR. Also, the scheduling effect is decomposed into a capital component and a return component. Section 5 shows that the later the payments are scheduled the greater is the AIRR. Section 6 is devoted to illustrating three real-life examples of PFI investments. Some concluding remarks end the paper.

2. Problems with the use of IRR as an indicator in relation to PFI schemes

The IRR is a commonly used indicator of the performance of PFI schemes in the UK. As guidance issued by the UK Treasury recognises, however, use of IRR is potentially misleading, unless the relevant payment streams are of a flat, annuity, type (Treasury, 2004). In Cuthbert and Cuthbert (2012) an appropriate indicator was developed for assessing how significant the departure from a flat payment stream might be in any particular case. In this section, we re-capitulate on the approach developed in that paper.

Public sector bodies which are commissioning PFI schemes have, of course, the problem of how they should assess the costs and benefits of the relevant projects. The advice given by the UK Treasury to such bodies is that they should basically rely on Net Present Values (NPVs). However, the Treasury recognises that measures based on IRR play an important part in PFI. The following quotation summarises the Treasury advice on the use of IRR:

> *The widespread use of IRRs in PFI projects reflects the generally even pattern of year-on-year operational cash flows in such projects. However, if a project has an uneven cash flow profile, the Authority should exercise great caution in using IRR as the basis of valuing investment in the project.* (Treasury, 2004).

As pointed out in Cuthbert and Cuthbert (2012), this Treasury advice makes sense. If there is indeed an even pattern of year on year cash flows in the particular payment stream being assessed (that is, if it is basically an annuity type payment stream), then knowledge of the initial capital investment and of the IRR (as well as of the investment's length) enables the NPV of the payment stream to be calculated at any desired discount rate. So there is essentially the same information content in knowing the IRR as in knowing the NPV profile. And even without the bother of working back from the IRR to the NPV, ranking schemes on the basis of IRRs will correspond to a ranking on NPVs if the projects to be compared have the same length, the same initial investment and flat cash flows.

If, however, the relevant payment profiles in PFI schemes are not of an annuity type, then the NPV of a given payment stream (at discount rates other than the IRR) is not determined by knowledge of the initial capital invested, the length and the IRR. Why does this matter?

Consider, for example, a typical PFI project (like the building and running of a hospital) from the point of view of the public sector client. During the construction phase, the public sector client makes no payments to the private sector consortium which is undertaking the project: but when the facility becomes operational, the public sector starts making regular unitary charge payments, which will go on during the 30 or so year life of the project. These unitary charge payments can be separated into two elements. The first of these, denoted in Cuthbert and Cuthbert (2012) as the "service element", covers the cost of ongoing activities relating to the operation and upkeep of the facility – like provision of contracted services, maintenance, and lifecycle costs. The second element, which Cuthbert and Cuthbert denoted the "non-service element" (NSE) covers loan charges, and pre-tax profits on equity.

The NSE is essentially what the public sector is paying, over a thirty year period, for the provision of the original capital asset: it may be, therefore, meaningful to work out the IRR of the payment stream which has, as initial negative terms, the original capital investment, and as subsequent positive terms (payments) the NSE element of the unitary charge. This is the project IRR, and is essentially the interest rate which the public sector is paying to fund the original capital investment under the assumption of a constant force of interest (see section 3). Since the private sector is taking on risk in undertaking the PFI project, and since the private sector in any event cannot borrow as cheaply as the public sector typically could, the project IRR will be higher than the cost of public sector borrowing. This is a well understood feature of PFI. (Note also that the project IRR in this case will be unique, because the payment stream is of the particular type which has negative terms preceding positive terms.)

However, while the project IRR is an interesting measure of cost, an even more important measure from the public sector viewpoint is the opportunity cost of the stream of payments which they have contracted to make in the form of the NSE: that is, how much could have been borrowed, at public sector interest rates, for the same cost as implied by the stream of NSE payments. This opportunity cost is appropriately measured by working out the NPV of the stream of NSE payments, discounted at an interest rate equal to the public sector borrowing rate. The important point is that, if the stream of NSE payments is not of a flat, annuity type, then it is no longer possible to work out the NPV of the NSE payment stream simply from knowledge of the original capital invested and the project IRR. So judgements about the opportunity cost of the project to the public sector which are founded only on the IRR could be hugely misleading.

The problem addressed in Cuthbert and Cuthbert's paper is, therefore, as follows. For a given discount rate less than the IRR of the project, how much of the NPV of the payment stream is due to the difference between the IRR and the discount rate (under the assumption that the profile of payments was of a flat, annuity type) and how much is due to the deviation of the actual payment profile from an annuity type profile?

The solution to this problem developed in Cuthbert and Cuthbert is as follows (see Cuthbert and Cuthbert, 2012, Appendix 1). Suppose we are examining a particular payment stream, $\mathbf{a} = (a_0, a_1, \ldots, a_n)$, which has IRR σ, so that $\mathrm{NPV}(\mathbf{a}, \sigma) = \sum_{j=0}^{n} a_j (1+\sigma)^{-j} = 0$, and we are interested in the NPV of $\mathbf{a}$ at a different discount rate r, that is, $r \neq \sigma$, $\mathrm{NPV}(\mathbf{a}, r) = \sum_{j=0}^{n} a_j (1+r)^{-j}$. Let $\mathbf{a}^-$ denote the vector consisting of the initial negative terms of $\mathbf{a}$ (with zeros elsewhere) and let $\mathbf{a}^+$denote the vector consisting of the positive repayment terms in $\mathbf{a}$ (with zeros elsewhere).

So $\mathbf{a} = \mathbf{a}^- + \mathbf{a}^+$ and $\mathbf{a}^- \leq 0 \;\; \mathbf{a}^+ \geq 0$; hence,

$$\mathrm{NPV}(\mathbf{a}) = \mathrm{NPV}(\mathbf{a}^-, r) + \mathrm{NPV}(\mathbf{a}^+, r)$$

where $\mathrm{NPV}(\mathbf{a}^-, r) < 0$ and $\mathrm{NPV}(\mathbf{a}^+, r) > 0$.

A ratio which is of particular interest is the ratio of the two component terms on the right hand side of equation (1) (adjusting the sign of $\mathrm{NPV}(\mathbf{a}^-, r)$ so that the ratio is positive),

$$\frac{\mathrm{NPV}(\mathbf{a}^+, r)}{|\mathrm{NPV}(\mathbf{a}^-, r)|}. \tag{2}$$

From the point of view of an investor incurring the outflow stream $\mathbf{a}^-$(costs) and receiving $\mathbf{a}^+$(benefits), equation (2) is a *benefit-cost* ratio. However, from the point of the public sector, $\mathbf{a}^-$ is the vector that represents the input of capital for initial construction made by the consortium, and the $\mathbf{a}^+$ vector represents the NSE payments by the public sector to the consortium; therefore, the ratio at (2) is a *cost-benefit* ratio and it is an appropriate measure of the opportunity cost to the public sector of making the stream of repayments $\mathbf{a}^+$, rather than the payment stream it would have made if it had borrowed at rate r.

Now suppose that, after the initial input of capital represented by $\mathbf{a}^-$, instead of paying out the actual repayment stream $\mathbf{a}^+$, an annual annuity style repayment, b, had been made, extending over the period from the year after the last capital input to the end of the contract: and also suppose that this annual payment, b, had been calculated so that the overall IRR is σ (that is the same as that of the original payment stream **a**). Let **b** consists of the original drawdowns of capital as initial negative terms, followed by the annuity repayments b as subsequent positive terms, that is, $\mathbf{b} = \mathbf{b}^- + \mathbf{b}^+$,where $\mathbf{b}^- = \mathbf{a}^-$ and $\mathbf{b}^+ = (0,0,\ldots,0,b,b,\ldots,b)$ and $\mathrm{IRR}(\mathbf{b}) = \mathrm{IRR}(\mathbf{a}) = \sigma$. (Note that we have departed slightly from the notation in Cuthbert and Cuthbert paper, for reasons which will become clear later.)

Then the cost-benefit ratio at (2) can be expressed in terms of the following identity:

$$\frac{\mathrm{NPV}(\mathbf{a}^+,r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|} = \frac{\mathrm{NPV}(\mathbf{b}^+,r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|} \cdot \frac{\mathrm{NPV}(\mathbf{a}^+,r)}{\mathrm{NPV}(\mathbf{b}^+,r)}. \tag{3}$$

The first term on the right hand side in formula (3), namely, $\frac{\mathrm{NPV}(\mathbf{b}^+,r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|}$, shows how much more (or less) it would have cost to fund the initial capital input by borrowing on an annuity rate scheme at interest rate σ, relative to borrowing on an annuity scheme at the chosen discount rate r: Cuthbert and Cuthbert called this the *interest component*.

The second term on the right hand side in formula (3), namely, $\frac{\mathrm{NPV}(\mathbf{a}^+,r)}{\mathrm{NPV}(\mathbf{b}^+,r)}$, shows how much more (or less) the NPV of the actual repayment scheme is, relative to the NPV of an annuity style repayment scheme with the same IRR σ: Cuthbert and Cuthbert called this the *scheduling component*, and a scheduling component which is materially different from 1 is indicative of a payment stream for which the assumption of flat, annuity style payments is violated.

In fact, the more the payments in the transaction vector **a** are shifted towards the later years in the life of the transaction, then the larger the scheduling effect: and, vice versa, the more the payments in the transaction vector **a** are shifted towards the earlier years in the life of the transaction, then the smaller the scheduling effect. We give a justification for these assertions in section 4, where we define a natural partial ordering on the space of transaction vectors, which corresponds to re-scheduling payments, and then show how the scheduling effect increases with this partial ordering.

The effect is that, when the scheduling component is less than 1, then this indicates that payments are, on average, advanced more towards the earlier years of the project than would occur with an annuity; while a scheduling component greater than 1 indicates payments which are deferred relative to annuity payments.

While it is not our purpose here to repeat in detail the empirical findings in the Cuthbert and Cuthbert paper, it is worth noting that, when the authors applied these techniques to the financial projections for eight PFI schemes, they found consistent, and in some cases, marked, deviations from flat payment schemes. The payments associated with the overall NSE tended to be slightly deferred (i.e., scheduling components slightly greater than 1); payments associated with senior debt interest were somewhat advanced, (i.e., scheduling components less than 1); and the payments associated with the equity returns on projects were markedly deferred (i.e., scheduling components much greater than 1).

An alternative decomposition into interest and scheduling effects can be defined as a profitability index:

$$\frac{\mathrm{NPV}(\mathbf{a},r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|}=\frac{\mathrm{NPV}(\mathbf{b},r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|}\cdot\frac{\mathrm{NPV}(\mathbf{a},r)}{\mathrm{NPV}(\mathbf{b},r)} \tag{4}$$

This alternative decomposition, (4), is essentially equivalent to the original decomposition, (3), used in the Cuthbert and Cuthbert paper. In particular:

- The term on the left hand side of (4) is equal to the term on the left hand side of (3), less 1: i.e.

$$\frac{\mathrm{NPV}(\mathbf{a},r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|}=\frac{\mathrm{NPV}(\mathbf{a}^+,r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|}-1.$$

- The interest component in (4) is equal to the interest component in (3), less 1: i.e.

$$\frac{\mathrm{NPV}(\mathbf{b},r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|}=\frac{\mathrm{NPV}(\mathbf{b}^+,r)}{|\mathrm{NPV}(\mathbf{a}^-,r)|}-1.$$

- The scheduling component in (4) is greater than or less than 1 according as the scheduling component in (3) is greater or less than 1.

The purpose of this paper is to expose the links between the decomposition into interest and scheduling components and the concept of AIRR, as developed by Magni (2010, 2013), and introduced in the next section. Since these links are more elegantly expressed using the decomposition in equation (4) rather than the one in equation (3), for the remainder of this paper we will work with interest and scheduling components as defined in equation (4).

3. The Average Internal Rate of Return (AIRR)

3.1 General definition of AIRR

Some of the difficulties encountered by the IRR as a notion of rate of return are well-known in the literature, while some others have only recently unearthed (see Magni, 2013, for a list of eighteen flaws). To overcome the IRR's difficulties, Magni (2010, 2013) developed a more general notion of rate of return, based on a capital-weighted mean of holding period rates, called *Average Internal Rate of Return* (AIRR). The AIRR approach consists in associating the capital amounts invested in each period with the corresponding period returns by means of a weighted arithmetic mean. Let $\mathbf{v}=(\mathrm{v}_0,\mathrm{v}_1,\dots,\mathrm{v}_{n-1})$ represent the capital invested in $\mathbf{a}$ at time $t=0,1,2,\dots,n-1$ with the initial condition $\mathrm{v}_0=-a_0$, and let v_n be the terminal capital (equal to zero after the last cash flows a_n has been received by the investor). The period rate of return in the interval $[j-1,j]$ is $i_j=(\mathrm{v}_j+a_j-\mathrm{v}_{j-1})/\mathrm{v}_{j-1}$. The capital-weighted mean

$$\mathrm{AIRR}(\mathbf{a},\mathbf{v},r)=\frac{i_1\mathrm{v}_0+i_2\mathrm{v}_1(1+r)^{-1}+\cdots+i_n\mathrm{v}_{n-1}(1+r)^{-(n-1)}}{\mathrm{v}_0+\mathrm{v}_1(1+r)^{-1}+\cdots+\mathrm{v}_{n-1}(1+r)^{-(n-1)}} \tag{5}$$

represents the overall rate of return associated with the capital stream **v**:taking time value of money into account, the numerator expresses the project's overall return and the denominator expresses the project's overall invested capital.

For the sake of notational simplicity, we will henceforth drop the dependence on r from the NPV symbols and the AIRR symbol, since r is assumed given.

Magni (2010, eq. (6)) showed that, for any vector **v** such that the initial condition $\mathrm{v}_0=-a_0$ is satisfied, the project NPV can be obtained as the product of the project invested capital $\mathrm{NPV}(\mathbf{v})=\sum_{j=0}^{n-1}\mathrm{v}_j(1+r)^{-j}$ and the project (excess) rate of return.

Proposition 1. For any capital vector **v,**

$$\mathrm{NPV}_1(\mathbf{a})=\mathrm{NPV}(\mathbf{v})\cdot(\mathrm{AIRR}(\mathbf{a},\mathbf{v})-r) \tag{6}$$

where $\mathrm{NPV}_1(\mathbf{a})\coloneqq\mathrm{NPV}(\mathbf{a})(1+r)$.

(See also Magni 2010, Theorem 2).

In essence, investing in project $\mathbf{a}$ boils down to investing the capital amount $\text{NPV}(\mathbf{v})$ at the rate $\text{AIRR}(\mathbf{a},\mathbf{v})$ while renouncing to investing the same capital at rate r.[5]

The AIRR approach guarantees NPV-consistency in the sense that the sign of the NPV is captured by the comparison of AIRR and cost of capital r: from (6),

$\text{NPV}(\mathbf{a}) > 0$ if and only if $\text{AIRR}(\mathbf{a},\mathbf{v}) > r$.

The reformulation of the NPV obtained in (6) highlights the contributions of both the capital component ($\text{NPV}(\mathbf{v})$) and the project's return component ($\text{AIRR}(\mathbf{a},\mathbf{v})$, net of cost of capital) to overall economic profitability. A given NPV can be obtained either as a result of investing a large capital at a small AIRR or as a result of investing a small capital at a high AIRR.

Therefore, the AIRR approach is a full substitute of NPV; but while NPV only supplies an overall measure economic value created, the AIRR approach decomposes it into the project investment scale and the project efficiency. This decomposition cannot be derived from a traditional NPV (or IRR) analysis. On the other hand, project ranking with NPV is more direct and simple: while it is possible to standardise AIRRs so as to rank projects correctly (Magni 2010, 2013), this crucially depends on there being a suitable or natural choice of a common capital base which makes the computation of standardised AIRRs meaningful and insightful. Equation (5) enables the analyst to make explicit use of the capital amounts v_t which are actually employed (i.e., those which represent the actual economic resources put in use by the investor). For example, consider a three-period project with holding period rates equal to $i_1 = 3\%, i_2 = 5\%, i_3 = 8\%$ and let $\text{v}_0 = 100, \text{v}_1 = 80, \text{v}_2 = 45$ be the capital values. Assuming $r = 4\%$, the project rate of return is

$$\text{AIRR}(\mathbf{a},\mathbf{v}) = \frac{0.03 \cdot 100 + 0.05 \cdot 80(1.04)^{-1} + 0.08 \cdot 45(1.04)^{-2}}{100 + 80(1.04)^{-1} + 45(1.04)^{-2}} = 0.0466$$

As the total invested capital is $\text{NPV}(\mathbf{v}) = 218.53$, the project consists of an overall investment of £218.53 at 4.66% return. As $4.66\% > 4\%$, the project NPV is positive, so value is created. In particular, $\text{NPV}(\mathbf{a}) = 218.53 \cdot (0.0466 - 0.04)/1.04 = 1.38$.[6]

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[5]In (6), one may redefine invested capital as being at beginning of period, rather than as at end of period: $\text{v}_0 = 0, \text{v}_1 = -a_0, \text{v}_{n+1} = 0$. This does not change the value of the AIRR and one gets $\text{NPV}(\mathbf{v}) = \sum_{j=1}^{n} \text{v}_j(1+r)^{-j}$ so that $\text{NPV}(\mathbf{a}) = \text{NPV}(\mathbf{v}) \cdot (\text{AIRR}(\mathbf{a},\mathbf{v}) - r)$. This structure is rather intuitive: it says that a project's NPV is the product of the excess return and invested capital. We have not adopted this particular convention in the paper, however, because the usual convention in finance is to consider capital invested as being at the end of the relevant period.

[6] One can unravel the cash flows from the definition of period rate of return: $a_t = \text{v}_{t-1}(1+i_t) - \text{v}_t$ so that $a_1 = 23, a_2 = 39, a_3 = 48.6$ and $a_0 = -\text{v}_0 = 100$. Discounting back, $\text{NPV}(\mathbf{a}) = -100 + 23(1.04)^{-1} + 39(1.04)^{-2} + 48.6(1.04)^{-3} = 1.38$.

A computational shortcut for the AIRR is directly obtained by (6):

$$\text{AIRR}(\mathbf{a},\mathbf{v}) = r + \frac{\text{NPV}_1(\mathbf{a})}{\text{NPV}(\mathbf{v})} \tag{7}$$

(in the example, $\text{AIRR}(\mathbf{a},\mathbf{v}) = 0.04 + 1.38(1.04)/218.53 = 0.0466$).

The author also showed that the IRR is but a particular case of (5) obtained when the interim values are derived from the IRR: let $\mathbf{v^a} = (\text{v}_0^\text{a}, \text{v}_1^\text{a}, \dots, \text{v}_{n-1}^\text{a})$ be such that $\text{v}_j^\text{a} = \text{v}_{j-1}^\text{a} \cdot (1+\sigma) - a_j$. In other words, this is the capital that would be employed at time j if the capital grew at a constant force of interest σ. Under this assumption, one gets $\text{AIRR}(\mathbf{a},\mathbf{v^a}) = \sigma$ so that (6) becomes

$$\text{NPV}_1(\mathbf{a}) = \text{NPV}(\mathbf{v^a}) \cdot (\sigma - r) \tag{8}$$

An IRR is then an *internal AIRR*.

3.2 The economic AIRR

Particularly meaningful is the case of the *economic* AIRR, that is, the rate of return that is generated in an efficient market, where current market prices fully reflect available information. Consider an investor (e.g., a firm), willing to undertake project **a**; how does an efficient market evaluate this situation if the equilibrium market rate is r? Before announcement of the project, shareholders' rate of return is r. When the firm announces the undertaking of investment **a**, there is a state of temporary disequilibrium and the stock price increases (or decreases) to arbitrage away the disequilibrium. This causes the firm's equity value to instantaneously change by $\text{v}_0^\text{e} - \text{v}_0$ where $\text{v}_0^\text{e} = \sum_{j=1}^n a_j(1+r)^{-j}$ is the economic (i.e., market) value of **a**, so $\text{v}_0^\text{e} - \text{v}_0$ is an instantaneous return to shareholders. Once the equilibrium is re-established again, shareholders' rate of return is, again, r, which implies that $\text{v}_j^\text{e} = \sum_{k=j+1}^n a_k(1+r)^{j-k}$ is the economic value of **a** at time $t \geq 1$ (see Magni, 2013). This means that the firm invests $\text{v}_0 = -a_0$ at time 0, v_1^e at time 1, v_2^e at time 2 and so on. Therefore, $\mathbf{V^e} = (\text{v}_0, \text{v}_1^\text{e}, \dots, \text{v}_{n-1}^\text{e})$ is the sequence of capital values invested in project **a** in the various periods.[7]

[7]Lindblom and Sjögren (2009) endorse the use of this sequence (which they call "strict market-based depreciation schedule") for increasing goal congruence in managerial performance evaluation. They show that such a choice is superior to ordinary straight-line, annuity-based or IRR-based depreciation schedules.

Let $\mathbf{a}^{\mathrm{e}} = (-\mathrm{v}_0^{\mathrm{e}}, a_1, \dots, a_n)$ be a modified cash-flow stream such that $\mathrm{v}_0^{\mathrm{e}}$ replaces v_0 and let $\mathbf{v}^{\mathbf{e}} = (\mathrm{v}_0^{\mathrm{e}}, \mathrm{v}_1^{\mathrm{e}}, \dots, \mathrm{v}_{n-1}^{\mathrm{e}})$ be its corresponding stream of economic values. Also, consider the incremental vectors $\mathbf{\Delta a}^{\mathrm{e}} = (\mathrm{a}_0 + \mathrm{v}_0^{\mathrm{e}}, 0, \dots, 0)$ and its corresponding capital stream $\mathbf{\Delta v}^{\mathrm{e}} = (\mathrm{v}_0 - \mathrm{v}_0^{\mathrm{e}}, 0, \dots, 0)$. Project **a** can be viewed as a portfolio of $\mathbf{a}^{\mathrm{e}}$ and $\mathbf{\Delta a}^{\mathrm{e}}$: that is, $\mathbf{a} = \mathbf{a}^{\mathbf{e}} + \mathbf{\Delta a}^{\mathrm{e}}$. Likewise, $\mathbf{V}^{\mathbf{e}} = \mathbf{v}^{\mathbf{e}} + \mathbf{\Delta v}^{\mathbf{e}}$(note that $\mathbf{\Delta a}^{\mathrm{e}} = -\mathbf{\Delta v}^{\mathrm{e}}$). By NPV additivity, this implies

$$\mathrm{NPV}(\mathbf{a}) = \mathrm{NPV}(\mathbf{a}^{\mathbf{e}}) + \mathrm{NPV}(\mathbf{\Delta a}^{\mathbf{e}}).$$

Definition 1. The *economic* AIRR (EAIRR) of project **a** is the AIRR that obtains in an efficient market, that is, the AIRR that results by picking $\mathbf{v} = \mathbf{V}^{\mathbf{e}}$.

Using (6), the EAIRR of **a** can be expressed as a weighted average of the constituents assets' rates of return, $\mathrm{AIRR}(\mathbf{a}^{\mathbf{e}}, \mathbf{v}^{\mathbf{e}})$ and $\mathrm{AIRR}(\mathbf{\Delta a}^{\mathrm{e}}, \mathbf{\Delta v}^{\mathrm{e}})$, respectively:

$$\mathrm{EAIRR} = \mathrm{AIRR}(\mathbf{a}, \mathbf{V}^{\mathrm{e}}) = \frac{\mathrm{AIRR}(\mathbf{a}^{\mathbf{e}}, \mathbf{v}^{\mathbf{e}}) \cdot \mathrm{NPV}(\mathbf{v}^{\mathbf{e}}) + \mathrm{AIRR}(\mathbf{\Delta a}^{\mathrm{e}}, \mathbf{\Delta v}^{\mathrm{e}}) \cdot \mathrm{NPV}(\mathbf{\Delta v}^{\mathbf{e}})}{\mathrm{NPV}(\mathbf{v}^{\mathbf{e}}) + \mathrm{NPV}(\mathbf{\Delta v}^{\mathbf{e}})}. \quad (9)$$

However, by (7),

$$\mathrm{AIRR}(\mathbf{\Delta a}^{\mathrm{e}}, \mathbf{\Delta v}^{\mathrm{e}}) = r + \frac{\mathrm{NPV}(\mathbf{\Delta a}^{\mathbf{e}})}{\mathrm{NPV}(\mathbf{\Delta v}^{\mathbf{e}})}(1+r) = r + \frac{a_0 + \mathrm{v}_0^{\mathrm{e}}}{\mathrm{v}_0 - \mathrm{v}_0^{\mathrm{e}}}(1+r) = -1.$$

This is intuitive, because $\mathbf{\Delta a}^{\mathbf{e}}$consists of one single cash flow: this means that one borrows $\mathrm{a}_0 + \mathrm{v}_0^{\mathrm{e}}$ and pays no interest nor reimburses the capital; in other words, the borrowing rate is -100% (which means that one is making money out of a borrowing: in financial jargon, it is an arbitrage). Further, by definition of economic value, the NPV of $\mathbf{a}^{\mathrm{e}}$ is zero: $\mathrm{NPV}(\mathbf{a}^{\mathbf{e}}) = 0$, which implies, by (7), that its AIRR is $\mathrm{AIRR}(\mathbf{a}^{\mathbf{e}}, \mathbf{v}^{\mathbf{e}}) = r$. Hence, project **a** is a portfolio of an equilibrium (i.e., zero NPV) asset yielding the equilibrium rate r and an incremental instantaneous return $\mathbf{\Delta a}^{\mathbf{e}}$ which represents the investor's arbitrage gain). This implies that the EAIRR is a weighted average of r and 100%:

$$\mathrm{EAIRR} = \frac{r \cdot \mathrm{NPV}(\mathbf{v}^{\mathbf{e}}) + 1 \cdot \mathrm{NPV}(|\mathbf{\Delta v}^{\mathbf{e}}|)}{\mathrm{NPV}(\mathbf{v}^{\mathbf{e}}) + \mathrm{NPV}(\mathbf{\Delta v}^{\mathbf{e}})}. \quad (10)$$

Considering that $\mathrm{NPV}(\mathbf{\Delta v}^{\mathbf{e}}) = \mathrm{v}_0 - \mathrm{v}_0^{\mathrm{e}} = -\mathrm{NPV}(\mathbf{a})$, (10) can be rewritten as

$$\mathrm{EAIRR} = r \cdot \frac{\mathrm{NPV}(\mathbf{v}^{\mathbf{e}})}{\mathrm{NPV}(\mathbf{V}^{\mathbf{e}})} + \frac{\mathrm{NPV}(\mathbf{a})}{\mathrm{NPV}(\mathbf{V}^{\mathbf{e}})} \quad (11)$$

which implies

$$\text{NPV}(\mathbf{a}) = \text{EAIRR} \cdot \text{NPV}(\mathbf{V^e}) - r \cdot \text{NPV}(\mathbf{v^e}). \tag{12}$$

Equation (12) says that the project's NPV represents an above-normal return; that is, it is the difference between the project's return[8] and the equilibrium return that investors obtain from the equilibrium asset $\mathbf{a^e}$.

Note that the period returns associated with the economic rate of return, $\text{AIRR}(\mathbf{V^e})$, are equal to the equilibrium rate, r, except in the first period, where the equilibrium asset's return is $r \cdot \text{v}_0^\text{e} = \text{v}_1^\text{e} + a_1 - \text{v}_0^\text{e}$ and the project's return is $i_1\text{v}_0 = \text{v}_1^\text{e} + a_1 - \text{v}_0$. Hence,

$$i_1\text{v}_0 = (\text{v}_0^\text{e} - \text{v}_0) + r\text{v}_0^\text{e}. \tag{13}$$

Equation (13) states that the return in the first period can be decomposed into two shares: the incremental amount $\text{v}_0^\text{e} - \text{v}_0$ and the first-period return of $\mathbf{a^e}$. This means that the first-period return is decomposed into an instantaneous return, $\text{NPV}(\mathbf{a})$, due to an immediate price increase, and an equilibrium return equal to $r \cdot \text{v}_0^\text{e}$, generated at time 1. Equation (13) summarises then the pricing behaviour of the efficient market: the first step is the instantaneous passage from v_0 to v_0^e, due to the efficient pricing process of the market which sweeps away the state of disequilibrium; the return gained in this step ('windfall gain') is just due to the price increase and so is equal to the NPV of **a**. The second step is the passage from v_0^e to v_1^e, which warrants a return equal to $\text{v}_1^\text{e} - \text{v}_0^\text{e} + a_1 = r \cdot \text{v}_0^\text{e}$. In the subsequent periods, the period rates of return of **a** and $\mathbf{a^e}$ are equal: $i_t = r$.This implies that the EAIRR can also be seen as a weighted average of the disequilibrium rate of return, i_1, and the equilibrium rate, r:

$$\text{EAIRR} = i_1 \cdot \frac{\text{v}_0}{\text{NPV}(\mathbf{V^e})} + r \cdot \frac{\text{NPV}(\mathbf{V^e}) - \text{v}_0}{\text{NPV}(\mathbf{V^e})} \tag{14}$$

To sum up, $\text{AIRR}(\mathbf{a^e}, \mathbf{v^e}) = r$ is the rate of return of an equilibrium asset $\mathbf{a^e}$ whereas the economic AIRR, represents the rate of return of a project **a** *as determined by the pricing process of an efficient market*.

4. Relationships between the scheduling effect and AIRR.

This section develops the links between the concepts introduced in the previous sections. In particular, we show how the scheduling effect defined in section 2 can be expressed, in two different ways, in terms of particular AIRRs.

[8] Note that $\text{EAIRR} \cdot \text{NPV}(\mathbf{v}) = i_1\text{v}_0 + \sum_{t=2}^{n} r \cdot \text{v}_{t-1}(1+r)^{-(t-1)}$.

4.1. The first expression

As in section 2, let **a** be a payment stream, where **a** consists of initial investment terms (negative terms), followed by succeeding repayments (non-negative terms): and let **a** have IRR equal to σ. Again as in section 1, let **b** be the vector with initial investment terms identical to those of **a**, followed by a stream of constant, i.e., annuity style, repayments b, where b is chosen so that the IRR of **b** is equal to σ.

Now, let $\mathbf{v^b}$ be defined, in relation to the payment stream **b** as the capital that would be invested if capital grew at the constant force of interest σ, so that

$$\mathrm{v}_0^{\mathrm{b}} = -b_0$$
$$\text{and } \mathrm{v}_j^{\mathrm{b}} = \mathrm{v}_{j-1}^{\mathrm{k}} \cdot (1+\sigma) - b_j, \quad j = 1, \dots, n. \tag{15}$$

Then,[9] as equation (6) holds for any **v**,

$$(1+r)\mathrm{NPV}(\mathbf{a}) = \mathrm{NPV}\big(\mathbf{v^b}\big) \cdot \big(\mathrm{AIRR}(\mathbf{a}, \mathbf{v^b}) - r\big) \tag{16}$$

where NPVs are calculated at discount rate r, and where $\mathrm{AIRR}(\mathbf{a}, \mathbf{v^b})$ denotes the AIRR of **a** calculated relative to $\mathbf{v^b}$ and discount rate r.

Further, from equation (8), applied to **b** rather than **a**, it follows that

$$(1+r)\mathrm{NPV}(\mathbf{b}) = \mathrm{NPV}\big(\mathbf{v^b}\big) \cdot (\sigma - r) \tag{17}$$

It follows immediately from (16) and (17) that, when $\sigma \neq r$ (as will always be the case in the situations in which we are interested),

$$\frac{\mathrm{NPV}(\mathbf{a})}{\mathrm{NPV}(\mathbf{b})} = \frac{\mathrm{AIRR}\big(\mathbf{a}, \mathbf{v^b}\big) - r}{\sigma - r}. \tag{18}$$

But the term on the left of this equation is just the scheduling effect in the decomposition into interest and scheduling effects given in equation (4). Equation (18), therefore, provides our first expression for the scheduling effect in terms of a particular AIRR: in this expression the relevant AIRR is the AIRR of **a** calculated relative to $\mathbf{v}^{\mathrm{b}}$ (and discount rate r), where $\mathbf{v^b}$ is, as seen, the vector of invested capital implied by the "annuity" payment vector **b** under the assumption of constant force of interest σ.

[9] Note that, if m is the date of the last outflow and $j \leq m$, then $\mathrm{v}_j^{\mathrm{a}} = \mathrm{v}_j^{\mathrm{b}}$, since $a_j = b_j$.

Note that equation (18) implies the following:

- the scheduling component is greater than 1 if, and only if, $\mathrm{AIRR}(\mathbf{v^b}) > \sigma$
- the scheduling component increases, (or decreases), the larger (or smaller) $\mathrm{AIRR}(\mathbf{a}, \mathbf{v^b})$ is relative to σ
- the interest component in eq. (4) decomposition is independent of $\mathrm{AIRR}(\mathbf{a}, \mathbf{v^b})$
- the NPV of **a** is greater than the NPV of **b** if, and only if, $\mathrm{AIRR}(\mathbf{a}, \mathbf{v^b}) > \sigma$.

Owing to (6) and (8), and considering that $\mathrm{AIRR}(\mathbf{a}, \mathbf{v^a}) = \sigma$, scheduling effect becomes

$$\frac{\mathrm{NPV}(\mathbf{a})}{\mathrm{NPV}(\mathbf{b})} = \frac{\mathrm{NPV}(\mathbf{v^a})/n}{\mathrm{NPV}(\mathbf{v^b})/n}. \tag{19}$$

Therefore, (19) indicates that a statistics based on average invested capital is a reliable indicator of the extent of departures of the relevant payment profiles from annuity type. That is, the scheduling effect can be alternatively captured by the ratio of the relative return component (eq. (18)) or the ratio of the relative capital component (eq. (19)).[10]

4.2. The second expression

In project finance transactions (and, in particular, in PFI schemes) the initial investment takes place over, typically, three or four years. The construction phase is where a lot of the risk associated with the project is located: and no income is received until construction is completed, and the unit becomes operational. During this phase, the project is not usually an attractive sale prospect in the secondary PFI market. It is usually only when construction is safely completed that the project becomes attractive to secondary investors, like pension funds, and is very often, at this stage, sold in the secondary PFI market, with a valuation indicating an implicit discount rate usually much lower than the project IRR. Hence, the idea of economic values just becomes compelling at the end of the construction phase, rather than at year 1. So, we now blend the capital stream $\mathbf{v^a}$ (which will be used for the construction phase), and the capital stream $\mathbf{V^e}$(which will be used for the payment phase) and show that this brings about an interesting decomposition of the scheduling component. To this end, let m the date at which the last outflow is incurred (end of the construction phase), and consider that **a** may be seen as a portfolio of two investments: the first one is the *construction* project $\mathbf{c} = (a_0^-, a_1^-, \dots, a_m^- + \mathrm{v}_m^{\mathrm{a}}, 0, \dots, 0)$ consisting of all the outflows a_t^- and the constructive sale of **c** at a price $\mathrm{v}_m^{\mathrm{a}}$; the second one is

[10] Obviously, n is irrelevant in eq. (19): to use total invested capital or average invested capital is the same: both measure the project investment scale.

the *payment* project $\mathbf{p} = (0, \dots, 0, -\mathrm{v}_m^{\mathrm{a}}, a_{m+1}^{+}, a_{m+2}^{+}, \dots, a_n^{+})$, consisting of the constructive purchase of $\mathbf{p}$ at $\mathrm{v}_m^{\mathrm{a}}$ and the payments a_t^{+} made by the public investor. Summing, $\mathbf{a} = \mathbf{c} + \mathbf{p}$. For the construction project, **c,** consider the sequence of internal values $\mathbf{v^c} = (\mathrm{v}_0^{\mathrm{a}}, \mathrm{v}_1^{\mathrm{a}}, \dots, \mathrm{v}_{m-1}^{\mathrm{a}}, 0, \dots, 0)$, which implies that the AIRR of **c** is σ; for the payment project, **p**, consider the sequence $\mathbf{v^p} = (0, \dots, 0, \mathrm{v}_m^{\mathrm{a}}, \mathrm{v}_{m+1}^{\mathrm{e}}, \mathrm{v}_{m+1}^{\mathrm{e}}, \dots, \mathrm{v}_{n-1}^{\mathrm{e}})$,[11] which implies that the AIRR of **p** is an EAIRR. Then, the sequence of capital values for **a** is

$$\mathbf{V_m^e} = \mathbf{v^c} + \mathbf{v^p} = (\mathrm{v}_0^{\mathrm{a}}, \mathrm{v}_1^{\mathrm{a}}, \dots, \mathrm{v}_m^{\mathrm{a}}, \mathrm{v}_{m+1}^{\mathrm{e}}, \mathrm{v}_{m+2}^{\mathrm{e}}, \dots, \mathrm{v}_{n-1}^{\mathrm{e}})$$

and the AIRR of **a,** relative to $\mathbf{V_m^e}$**,** is a weighted average of the two rates of return:

$$\mathrm{AIRR}(\mathbf{a}, \mathbf{V_m^e}) = \sigma \cdot \frac{\mathrm{NPV}(\mathbf{v^c})}{\mathrm{NPV}(\mathbf{V_m^e})} + \mathrm{EAIRR^p} \cdot \frac{\mathrm{NPV}(\mathbf{v^p})}{\mathrm{NPV}(\mathbf{V_m^e})} \tag{20}$$

where $\mathrm{EAIRR^p}$ now refers to **p** rather than **a.**

Formally, the EAIRR of **a** is a particular case of (20), obtained by picking $m = 0$: $\mathrm{AIRR}(\mathbf{a}, \mathbf{V_0^e}) = \mathrm{AIRR}(\mathbf{a}, \mathbf{V^e}) = \mathrm{EAIRR}$. Analogously, the IRR of **a** is a particular case of (20), obtained by picking $m = n - 1$: $\mathrm{AIRR}(\mathbf{a}, \mathbf{V_{n-1}^e}) = \mathrm{AIRR}(\mathbf{a}, \mathbf{v^a}) = \mathrm{IRR}$. We call (20) *blended economic* AIRR (BEAIRR).

Obviously, using (7), the BEAIRR can also be computed with the shortcut

$$\mathrm{BEAIRR} = r + \frac{\mathrm{NPV}_1(\mathbf{a})}{\mathrm{NPV}(\mathbf{V_m^e})}. \tag{21}$$

Using the BEAIRR, the scheduling component is clearly affected by both rate and capital. More precisely, from (6),

$$\frac{\mathrm{NPV}(\mathbf{a}, r)}{|\mathrm{NPV}(\mathbf{a}^-, r)|} = \frac{\mathrm{NPV}(\mathbf{b}, r)}{|\mathrm{NPV}(\mathbf{a}^-, r)|} \cdot \frac{\mathrm{NPV}(\mathbf{V_m^e}) \cdot (\mathrm{BEAIRR} - r)}{\mathrm{NPV}(\mathbf{v^b}) \cdot (\sigma - r)}.$$

Therefore, we obtain a decomposition of the scheduling component into a (relative) capital component and a (relative) return component:

[11] Note that $\mathbf{v^p}$ fulfils the equality condition between the first capital and the investment cost $(-\mathrm{v}_m^{\mathrm{b}})$ changed in sign.

$$\frac{\text{NPV}(\mathbf{a},r)}{\text{NPV}(\mathbf{b},r)} = \overbrace{\frac{\text{NPV}(\mathbf{V}_m^{\mathbf{e}})}{\text{NPV}(\mathbf{v}^{\mathbf{b}})}}^{\text{capital component}} \cdot \overbrace{\frac{\text{BEAIRR} - r}{\sigma - r}}^{\text{return component}}. \tag{22}$$

The interaction between capital and rate determines whether a scheduling effect arises or not and the magnitude of it. In particular, letting $K \coloneqq \frac{\text{NPV}(\mathbf{V}_m^{\mathbf{e}})}{\text{NPV}(\mathbf{v}^{\mathbf{b}})}$ be the (relative) capital component and $R \coloneqq \frac{\text{BEAIRR}-r}{\sigma-r}$ be the (relative) return component, (22) can be written as $\frac{\text{NPV}(\mathbf{a},r)}{\text{NPV}(\mathbf{b},r)} = K \cdot R$. While K tells us by how much the capital component of **a** exceeds the capital component of **b**, R tells how by how much the return component of **a** exceeds the return component of **b**. Evidently, the scheduling effect arises if and only if $K \cdot R \neq 1$.

To sum up, one may conveniently decompose the scheduling component into a relative capital component and a relative return component. This enables the evaluator to understand not only the magnitude of the scheduling effect, but also how the individual components K and R interact in producing it:

Profitability index

$$\frac{\text{NPV}(\mathbf{a})}{|\text{NPV}(\mathbf{a}^-)|}$$

Interest component

$$\frac{\text{NPV}(\mathbf{b})}{|\text{NPV}(\mathbf{a}^-)|}$$

Scheduling component

$$\frac{\text{NPV}(\mathbf{a})}{\text{NPV}(\mathbf{b})}$$

Capital component

$$K = \frac{\text{NPV}(\mathbf{V}_m^{\mathbf{e}})}{\text{NPV}(\mathbf{v}^{\mathbf{b}})}$$

Return component

$$R = \frac{\text{BEAIRR}-r}{\sigma-r}$$

Note that, in equation (18), the relative capital component is nullified, as we have used $\mathbf{v^b}$ as the capital stream for $\mathbf{a}$, so the scheduling effect is entirely captured by the relative return component. Analogously, in equation (19), the relative return component is nullified, as we have used $\mathbf{v^a}$ as the capital stream for $\mathbf{a}$, so the scheduling effect is entirely captured by the relative capital component. Unlike (18) and (19), the use of BEAIRR in (22) recognises both effects.

5. A partial ordering on the set of transaction vectors which has a simple relationship with the scheduling effect and the AIRR.

In section 1, we defined the scheduling component as $\frac{\text{NPV}(\mathbf{a})}{\text{NPV}(\mathbf{b})}$, and showed how it has a simple expression in terms of a particular AIRR. What we did not do in detail, however, was to justify the claim we had made there, that the more the repayment terms in the transaction vector **a** were shifted towards the later years in the life of the transaction, then the larger the scheduling effect. In this section, we repair this gap, by defining a natural partial ordering on the space of transaction vectors, which corresponds to re-scheduling payments, and then showing how the scheduling effect increases with this partial ordering.

Let **x** and **y** be transaction vectors, with the same IRR σ, and let $\mathbf{z} = \mathbf{x} - \mathbf{y}$.

Definition 2. $\mathbf{x} \succ\succ \mathbf{y}$ if, and only if, there exists an integer k such that $z_j \leq 0$ for all $j \leq k$ and $z_j \geq 0$ for all $j > k$.

Conversely, the relationship $\prec\prec$ is defined by $\mathbf{x} \prec\prec \mathbf{y}$ if and only if $\mathbf{y} \succ\succ \mathbf{x}$.

Now define the relationship $\mathbf{x} \succ \mathbf{y}$ between two transaction vectors (again with the same IRR σ) as follows.

Definition 3. $\mathbf{x} \succ \mathbf{y}$ if, and only if, there exist transaction vectors $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \ldots, \boldsymbol{\alpha}_m$, for some m, where each of the $\boldsymbol{\alpha}_j$ has IRR σ, such that
$\mathbf{x} \succ\succ \boldsymbol{\alpha}_1 \succ\succ \boldsymbol{\alpha}_2 \succ\succ \cdots \succ\succ \boldsymbol{\alpha}_m \succ\succ \mathbf{y}$.

The relationship $\succ$ is a partial ordering on the set of transaction vectors with IRR σ: that is, it satisfies the relationships

$\mathbf{x} \succ \mathbf{y}$ and $\mathbf{y} \succ \mathbf{z} \Rightarrow \mathbf{x} \succ \mathbf{z}$,

$\mathbf{x} \succ \mathbf{x}$,

if $\mathbf{x} \succ \mathbf{y}$ and $\mathbf{y} \succ \mathbf{x}$, then $\mathbf{x} = \mathbf{y}$.

The converse relationship $\prec$ is defined by $\mathbf{x} \prec \mathbf{y}$ if and only if $\mathbf{y} \succ \mathbf{x}$.

If $\mathbf{x} \succ \mathbf{y}$, therefore, the payments in transaction $\mathbf{x}$ are scheduled to be later than those in $\mathbf{y}$, in the sense defined in Definition 3.

Given the way the relationship $\succ$ has been defined, it is not immediately obvious, for any specific pair of transaction vectors $\mathbf{x}$ and $\mathbf{y}$, whether the relationship $\succ$ holds between them. The following theorem gives a necessary and sufficient condition for the relationship $\succ$ to hold.

It is necessary to introduce some notation first. If $\mathbf{z}$ is a transaction vector with IRR σ, let the vector $\mathbf{v^z}$ be defined, in relation to the transaction vector $\mathbf{z}$, as the capital that would be invested if capital grew at the constant force of interest σ, so that

$$\mathrm{v}_0^{\mathrm{z}} = -z_0,$$

$$\mathrm{v}_j^{\mathrm{z}} = (1+\sigma)\mathrm{v}_{j-1}^{\mathrm{z}} - z_j \qquad j = 1,2,\dots,n \tag{23}$$

If $\mathrm{v}_j^{\mathrm{z}} \geq 0$ for all j, then we denote $\mathbf{z}$ as a *Soper* transaction, since this type of transaction was considered in Soper (1959).[12]

We state, without proof, the following standard facts about Soper transactions, which are not difficult to establish:

i. If $\mathbf{z}$ is a transaction where initial non-positive terms are followed by subsequent non-negative terms, then it is Soper.
ii. The IRR of a Soper transaction is unique.
iii. If $\mathbf{z}$ is a Soper transaction with IRR σ, then $\mathrm{NPV}(\mathbf{z}, r) > 0$ for all discount rates $r < \sigma$.

We characterise the relationship $\succ$ by the following theorem.

Proposition 2. If $\mathbf{x}$ and $\mathbf{y}$ are transaction vectors with the same IRR, and if $\mathbf{z} = \mathbf{x} - \mathbf{y}$, then $\mathbf{x} \succ \mathbf{y}$ if, and only if, $\mathbf{z}$ is a Soper transaction.

Proof.

(i) Suppose $\mathbf{x} \succ \mathbf{y}$. Let $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \dots, \boldsymbol{\alpha}_m$ be the transaction vectors whose existence is implied by Definition 3. Then, $\mathbf{z} = \mathbf{x} - \mathbf{y} = (\mathbf{x} - \boldsymbol{\alpha}_1) + (\boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2) + \cdots + (\boldsymbol{\alpha}_m - \mathbf{y})$. Now, each of the bracketed terms in this sum has initial non-positive terms followed by subsequent non-negative terms, and is therefore a Soper transaction.

[12] More precisely, Soper (1959) considered the condition $\mathrm{v}_j^{\mathrm{z}} > 0$. The extension to the general case $\mathrm{v}_j^{\mathrm{z}} \geq 0$ can be found in Gronchi (1986).

Since it is easily established that a sum of Soper transactions with the same IRRs is also Soper, it follows that $\mathbf{z}$ is Soper.

(ii) Suppose $\mathbf{z}$ is Soper, with IRR σ. Then, $\mathbf{v^z} \geq \mathbf{0}$ .Now let β_j, for $j = 0,1, \dots, n-1$ be defined as the transaction vector which has j-th term equal to $-\mathrm{v}_j^{\mathrm{z}}$, $(j+1)$-th term equal to $(1+\sigma)\mathrm{v}_j^{\mathrm{z}}$, and all other terms zero. Now since, from equation (23), $z_j = (1+\sigma)\mathrm{v}_{j-1}^{\mathrm{z}} - \mathrm{v}_j^{\mathrm{z}}$, it follows that $\mathbf{z} = \sum_{j=0}^{n-1} \beta_j$.Hence, $\mathbf{y} \prec\prec \mathbf{y} + \boldsymbol{\beta}_\mathbf{0} \prec\prec \mathbf{y} + \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 \prec\prec \dots \prec\prec \mathbf{y} + \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 + \cdots + \boldsymbol{\beta}_{n-1} = \mathbf{y} + \mathbf{z} = \mathbf{x},$ which implies $\mathbf{x} \succ \mathbf{y}$. This completes the proof.

Corollary. If $\mathbf{x}$ and $\mathbf{y}$ are transaction vectors, with the same IRR σ, and if $\mathbf{x} \succ \mathbf{y}$, then $\mathrm{NPV}(\mathbf{x}, r) \geq \mathrm{NPV}(\mathbf{y}, r)$ for all $r < \sigma$, and the inequality is strict unless $\mathbf{x} = \mathbf{y}$.

Proof. It follows from Proposition 2 that $\mathbf{x} = \mathbf{y} + \mathbf{z}$, where $\mathbf{z}$ is Soper. The corollary then follows since, for $\mathbf{z}$ Soper, $\mathrm{NPV}(\mathbf{x}, r) > 0$ for all discount rates $r < \sigma$.

We now return to the situation considered in section 1 of this paper, and show how the scheduling effect defined there relates to the partial ordering $\prec$.

More specifically, let $\mathbf{a}_1$ and $\mathbf{a}_2$ be transaction vectors of the type considered in section 1: that is, having initial investment (non-positive) terms followed by non-negative terms. Suppose that both vectors have the same IRR σ, and that the initial non-positive terms in both vectors are the same. Again as in section 1, let $\mathbf{b}$ be the vector with initial investment terms identical to those of $\mathbf{a}_1$ (and $\mathbf{a}_2$), followed by a stream of constant, i.e., annuity style, repayments b, where b is chosen so that IRR of $\mathbf{b}$ is equal to σ.

Then we have the following:

Proposition 3. (*Relationship between the scheduling effect and the partial ordering*).

If $\mathbf{b} \prec \mathbf{a}_1 \prec \mathbf{a}_2$, then $1 \leq \frac{\mathrm{NPV}(\mathbf{a}_1, r)}{\mathrm{NPV}(\mathbf{b}, r)} \leq \frac{\mathrm{NPV}(\mathbf{a}_2, r)}{\mathrm{NPV}(\mathbf{b}, r)}$ for all $r < \sigma$, and the inequalities are strict unless the relevant transactions are identical.

If $\mathbf{a}_1 \prec \mathbf{a}_2 \prec \mathbf{b}$, then $\frac{\mathrm{NPV}(\mathbf{a}_1, r)}{\mathrm{NPV}(\mathbf{b}, r)} \leq \frac{\mathrm{NPV}(\mathbf{a}_2, r)}{\mathrm{NPV}(\mathbf{b}, r)} \leq 1$ for all $r < \sigma$, and the inequalities are strict unless the relevant transactions are identical.

Proof. It follows immediately from the corollary to Proposition 2, on noting that, since $\mathbf{b}$ is Soper, $\mathrm{NPV}(\mathbf{b}, r) > 0$ for all $r < \sigma$.

In other words, the scheduling effect defined in section 1 increases monotonically with the partial ordering $\prec$.

Because of equation (18) in section 3, it also follows that there is a relationship between the partial ordering and the AIRR, in the following sense. If we denote, in an obvious notation, $\text{AIRR}(\mathbf{a}, \mathbf{v}^{\mathbf{b}}, r)$ as being the AIRR of **a** relative to the capital invested in **b** (at constant force of interest σ), and relative to discount rate r, then the following result holds.

Proposition 4. (*Relationship between AIRR and the partial ordering*).
If $\mathbf{b} \prec \mathbf{a}_1 \prec \mathbf{a}_2$, then $\sigma \leq \text{AIRR}(\mathbf{a}_1, \mathbf{v}^{\mathbf{b}}, r) \leq \text{AIRR}(\mathbf{a}_2, \mathbf{v}^{\mathbf{b}}, r)$ for all $r < \sigma$, and the inequalities are strict unless the relevant transactions are identical.
If $\mathbf{a}_1 \prec \mathbf{a}_2 \prec \mathbf{b}$, then $\text{AIRR}(\mathbf{a}_1, \mathbf{v}^{\mathbf{b}}, r) \leq \text{AIRR}(\mathbf{a}_2, \mathbf{v}^{\mathbf{b}}, r) \leq \sigma$ for all $r < \sigma$, and the inequalities are strict unless the relevant transactions are identical.

Proof. It follows immediately from applying equation (18) to the previous result.

Of course, the relationship $\succ$ which we have considered here is only a partial ordering on the set of transaction vectors: so it will not necessarily hold in practice that either $\mathbf{a} \prec \mathbf{b}$ or $\mathbf{b} \prec \mathbf{a}$. Nevertheless, the partial ordering is interesting, because it does illustrate how the scheduling effect and the corresponding AIRR increase monotonically with the kind of rescheduling of payments which increases the partial order. This gives a good intuitive justification for our use of the term 'scheduling component' in section 1.

6. Some numerical examples

In this section, we illustrate the application of the above theory by means of three numerical examples. The data in these examples have been taken from the financial projections for actual PFI schemes. In each case, the projects considered are PFI hospital schemes in Scotland and the North of England on which construction commenced in the late 1990s, and the relevant data were obtained using requests under United Kingdom Freedom of Information Acts.

The three examples have been chosen to illustrate different aspects of the PFI process.

Examples 1-3 represent the perspective of investors in different aspects of the financing of the project. Examples 1 and 2 represent cases of investors incurring costs, and then receiving benefits. In the first example, the negative terms of **a** represent investment of senior debt in a project, and the positive terms represent the returns to the investor by way of repayments of capital and of interest. The second example represents the perspective of an equity investor in a project: in this case, the negative terms of $\mathbf{a}$ are the inputs of subordinate debt and equity to the project, and the positive terms are the outputs of this investment distributed as dividends.

Example 3 illustrates the perspective of the public sector client of a PFI project. The negative terms in the payment stream in this case represent the initial investment of capital by the private sector consortium in building and equipping the hospital: and the positive terms represent the non-service element (NSE) of the unitary charge paid each year to the consortium during the operational years of the project, where the NSE is as defined in section 1. From this perspective, the negative terms represent the benefits received by the public sector, in terms of the provision of the hospital facility, and the positive terms the subsequent costs incurred in paying for these benefits.

In examples 1 and 3, the choice of reference interest rate (cost of capital) for calculating NPVs is 5%. In example 2, a discount rate of 9% was taken: this is appropriate, because it is close to the return investors would have assumed at the time when investing in PFI equity in the secondary PFI market.

The first example is illustrated in Table 1,where **b** contains the cash flows debtholders would receive if they invested in an annuity-repayment loan with IRR equal to the IRR of **a.** Looking first of all at the first expression of the scheduling effect in terms of AIRR, as developed in Section 3.1, the AIRR of $\mathbf{a}$ (relative to $\mathbf{v^b}$) is $\text{AIRR}(\mathbf{a}, \mathbf{v^b}) = 6.72\%$, whereas the IRR signals a greater rate of return of 7.18%. Since the AIRR is less than the IRR, equation (18) implies that there will be a scheduling effect which is less than 1: in fact, from (18), the magnitude of the scheduling effect is 0.789. The same result can be obtained from the ratio of the (average or total) invested capital of $\mathbf{a}$ and **b:** $21{,}007.19/26{,}636.14 = 0.789$ (see equation (19)). It is easy to see that the interest component is 0.317, which means that the profitability index is $0.317 \cdot 0.789 = 0.25$, i.e., debtholders of the project company increase their wealth by £0.25 per pound invested. It is worth noting that $\text{AIRR}(\mathbf{a}, \mathbf{v^b})$, while smaller than the IRR, is greater than the required return to debt ($r = 5\%$). This implies that the project financing transaction is a value-creation project for debtholders, but the latter would be better off by investing their funds in **b** (if feasible), that is, in a level-payment investment with the same IRR, σ.

Turning now to the second expression for the scheduling effect, as developed in Section 3.2, which involves estimating the interim values according to market values (after completion of the hospital), then the rate of return to debtholders is measured by $\text{BEAIRR} = 6.97\%$. The scheduling component can then be decomposed in relative capital component and relative return component: $K = 0.871,\ R = 0.905$ so that $K \cdot R = 0.789$.

It is of interest to note that, in this particular example, $\mathbf{z} = \mathbf{b} - \mathbf{a}$ is a Soper project ($\mathbf{v^b} - \mathbf{v^a} \geq 0$ for all t), so, by Proposition 2, the partial ordering $\mathbf{a} \prec \mathbf{b}$ holds, which means $\text{AIRR}(\mathbf{a}, \mathbf{v^b}) < \sigma$, as seen. Indeed, the stronger relationship $\mathbf{a} \prec\prec \mathbf{b}$ also holds.

In the equity example (Table 2), $\text{AIRR}(\mathbf{a}, \mathbf{v^b}) = 35.6\%$ is considerably greater than the IRR, $\sigma = 23.2\%$, which, by formula (18), implies a scheduling component much greater than 1: in fact, formula (18) gives an actual value for the scheduling component of 1.88, and is just equal to the ratio of the average (or total) capital values: $\text{NPV}(\mathbf{v^a})/\text{NPV}(\mathbf{v^b}) = 1.876$. Decomposition of the scheduling component is possible if BEAIRR if used. In this case, the BEAIRR is equal to the EAIRR, for all cash flows are positive after the initial equity contribution: $\text{BEAIRR} = \text{EAIRR} = 14.64\%$. Decomposing the scheduling component, it is clear that the capital component $K = 4.72$ plays an important role in the high value of the scheduling component.

In this example, $\mathbf{z} = \mathbf{a} - \mathbf{b}$ is a Soper project ($\mathbf{v^a} - \mathbf{v^b} \geq 0$ for all t), so the relation $\mathbf{a} \succ \mathbf{b}$ holds. Indeed, the stronger relationship $\mathbf{a} \succ\succ \mathbf{b}$ also holds.

As for the third example, one should bear in mind that negative (positive) cash flows are benefits (costs) for the public sector, so the AIRRs and the IRR are borrowing rates and the profitability index is not a profitability measure but a *cost* measure. As can be seen from Table 3, $\text{AIRR}(\mathbf{a}, \mathbf{v^b}) = 7.75\%$ is smaller than the IRR, $\sigma = 7.93\%$, suggesting that the public sector benefits from turning to the project financing transaction instead of borrowing with a level-payment annuity-like financing, even though the IRR is the same. The *cost* index is 0.35, which means that the public sector pays £0.35 pounds for each pound borrowed and there is a slight scheduling effect.

Note that, for this last example, $\mathbf{z} = \mathbf{b} - \mathbf{a}$ is a Soper project, so the relations $\mathbf{a} \prec \mathbf{b}$ holds. In this case, the stronger relation $\mathbf{a} \prec\prec \mathbf{b}$ holds.

7. Concluding remarks

This paper illustrates the danger in relying on IRR as an indicator to assess the performance of financial projects. The use of IRRs should be discouraged, as other metrics are available that more properly adhere to the underlying economic referents.,. So to avoid the danger of being seriously misled, the analyst should not rely on IRR, but should base conclusions on appropriate deeper analyses. This paper shows that a possible way of coping with this issue is to rest on the notion of scheduling effect as developed in Cuthbert and Cuthbert (2012) and make appropriate use of Magni (2010, 2013)'s AIRR approach, following the equivalences developed in this paper. An appropriate analysis should also require quoting average outstanding capital values in association with IRRs or AIRRs.

While the original paper by Cuthbert and Cuthbert was concerned with a particular class of financial project (namely, PFI schemes), the techniques developed there, and the extension of these techniques in this paper using AIRRs, are actually of much more general applicability. In the case of PFI schemes, the UK Treasury identified the inadequacy of IRR alone as an indicator if payment streams were not of the flat, annuity, type: so once it was established empirically that payment streams are commonly not flat in PFI schemes, there was a clear need to develop further indicators – which was the impetus behind the Cuthbert and Cuthbert paper. But it is also clear that, in general investment problems, there will often be similar requirements. And it will also commonly be the case that payment profiles will not be of an annuity type (or cannot be assumed to be of this type).

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Table 1a. Senior debt – cash flows and capital values

year	cash flows		capital values		
	a	**b**	$\mathbf{v^a}$	$\mathbf{v^b}$	$\mathbf{V_4^e}$
0	−3,497.46	−3,497.46	3,497.46	3,497.46	3,497.46
1	−18,602.57	−18,602.57	22,351.07	22,351.07	22,351.07
2	−23,780.31	−23,780.31	47,735.69	47,735.69	47,735.69
3	−17,360.83	−17,360.83	68,522.89	68,522.89	68,522.89
4	−1,739.38	−1,739.38	75,180.69	75,180.69	75,180.69
5	6,865.12	6,167.10	73,711.87	74,409.89	87,612.29
6	6,952.52	6,167.10	72,050.22	73,583.77	85,040.38
7	7,016.40	6,167.10	70,205.42	72,698.36	82,276.00
8	7,056.94	6,167.10	68,187.67	71,749.38	79,332.86
9	7,158.24	6,167.10	65,923.79	70,732.30	76,141.26
10	7,289.95	6,167.10	63,365.70	69,642.21	72,658.37
11	7,356.37	6,167.10	60,557.58	68,473.88	68,934.93
12	7,450.75	6,167.10	57,453.52	67,221.68	64,930.92
13	7,610.31	6,167.10	53,967.10	65,879.61	60,567.16
14	7,752.05	6,167.10	50,088.69	64,441.20	55,843.47
15	7,937.77	6,167.10	45,746.17	62,899.55	50,697.88
16	8,123.28	6,167.10	40,906.45	61,247.25	45,109.49
17	6,902.18	6,167.10	36,940.45	59,476.34	40,462.79
18	6,889.27	6,167.10	32,702.69	57,578.33	35,596.66
19	6,938.59	6,167.10	28,111.43	55,544.07	30,437.91
20	6,984.53	6,167.10	23,144.67	53,363.81	24,975.27
21	7,029.10	6,167.10	17,776.84	51,027.05	19,194.93
22	7,065.24	6,167.10	11,987.59	48,522.56	13,089.44
23	7,097.52	6,167.10	5,750.51	45,838.30	6,646.40
24	1,206.63	6,167.10	4,956.64	42,961.38	5,772.08
25	377.50	6,167.10	4,934.91	39,877.95	5,683.19
26	377.50	6,167.10	4,911.63	36,573.21	5,589.84
27	377.50	6,167.10	4,886.67	33,031.26	5,491.84
28	377.50	6,167.10	4,859.93	29,235.07	5,388.93
29	377.50	6,167.10	4,831.26	25,166.40	5,280.88
30	377.50	6,167.10	4,800.54	20,805.69	5,167.42
31	377.50	6,167.10	4,767.61	16,131.99	5,048.29
32	377.50	6,167.10	4,732.32	11,122.81	4,923.20
33	377.50	6,167.10	4,694.50	5,754.08	4,791.86
34	5,031.46	6,167.10	0.00	0.00	0.00
NPV	14,813.95	18,783.4	714,244.4	905,628.9	788,864.6

Table 1b. Senior debt – scheduling component

AIRR			
IRR (σ)	7.18%	$\lvert\text{NPV}(\mathbf{a}^{-})\rvert$	59,211.6[13]
COC (r)	5%	$\text{NPV}(\mathbf{b})$	18,783.4
$\text{AIRR}(\mathbf{a}, \mathbf{v}^{\mathbf{b}})$	6.72%	interest component	0.317
scheduling component			
eq. (18)	0.789		
Average capital			
$\text{NPV}(\mathbf{v}^{\mathbf{a}})/34$	21,007.19	Profitability index	
$\text{NPV}(\mathbf{v}^{\mathbf{b}})/34$	26,636.14	$\text{NPV}(\mathbf{a})/\lvert\text{NPV}(\mathbf{a}^{-})\rvert$	0.25
scheduling component			
eq. (19)	0.789		
BEAIRR			
$\text{AIRR}(\mathbf{a}, \mathbf{V}_{\mathbf{4}}^{\mathbf{e}})$	6.97%		
capital component	0.871		
return component	0.905		
scheduling component			
eq. (22)	0.789		

[13] We remind that $\text{NPV}(\mathbf{a}^{-})$ is defined as the present value, computed at r, of the project's negative cash flows.

Table 2a. Equity – cash flows and capital values

year	cash flows		capital values		
	a	**b**	$\mathbf{v^a}$	$\mathbf{v^b}$	$\mathbf{V_0^e}$
0	−8400.1	−8400.1	8400.1	8400.1	8400.10
1	1560.8	1951.9	8,788.2	8,397.1	31,541.37
2	1584.6	1951.9	9,242.5	8,393.3	32,795.46
3	1584.6	1951.9	9,802.2	8,388.7	34,162.41
4	1584.6	1951.9	10,491.7	8,383.1	35,652.39
5	1584.6	1951.9	11,341.2	8,376.1	37,276.46
6	1987.7	1951.9	11,984.8	8,367.5	38,643.66
7	2192.4	1951.9	12,572.9	8,356.9	39,929.17
8	2280.1	1951.9	13,209.8	8,343.8	41,242.67
9	2351.1	1951.9	13,923.5	8,327.8	42,603.38
10	2407.0	1951.9	14,746.9	8,307.9	44,030.69
11	2460.9	1951.9	15,707.4	8,283.5	45,532.57
12	3180.8	1951.9	16,170.8	8,253.5	46,449.66
13	2246.1	1951.9	17,676.4	8,216.4	48,384.03
14	2183.3	1951.9	19,594.1	8,170.8	50,555.25
15	2206.9	1951.9	21,933.2	8,114.5	52,898.32
16	2216.8	1951.9	24,805.1	8,045.2	55,442.34
17	2230.6	1951.9	28,329.5	7,959.9	58,201.53
18	2237.1	1951.9	32,665.0	7,854.7	61,202.52
19	4819.0	1951.9	35,424.5	7,725.1	61,891.71
20	8860.8	1951.9	34,782.5	7,565.5	58,601.13
21	9089.3	1951.9	33,763.0	7,368.9	54,785.96
22	9478.6	1951.9	32,117.7	7,126.6	50,238.08
23	9323.3	1951.9	30,245.9	6,828.1	45,436.17
24	9516.5	1951.9	27,746.7	6,460.3	40,008.90
25	9668.1	1951.9	24,516.0	6,007.3	33,941.57
26	8937.4	1951.9	21,266.5	5,449.1	28,058.90
27	5422.2	1951.9	20,778.3	4,761.4	25,161.99
28	7371.0	1951.9	18,228.0	3,914.1	20,055.55
29	14742.3	1951.9	7,714.8	2,870.3	7,118.26
30	7758.9	1951.9	1,745.7	1,584.3	8,928.52
31	2150.8	1951.9	0.0	0.0	0.00
NPV	22,117.6	11,788.2	169,766.1	90,482.0	427,337.7

Table 2b. Equity – scheduling component

AIRR			
IRR (σ)	23.2%	$\lvert\mathrm{NPV}(\mathbf{a}^-)\rvert$	8,400.10
COC (r)	9%	$\mathrm{NPV}(\mathbf{b})$	11,788.2
$\mathrm{AIRR}(\mathbf{a}, \mathbf{v^b})$	35.64%	interest component	1.403
scheduling component			
eq. (18)	1.876		
Average capital			
$\mathrm{NPV}(\mathbf{v^a})/31$	5,476.33	Profitability index	
$\mathrm{NPV}(\mathbf{v^b})/31$	2,918.77	$\mathrm{NPV}(\mathbf{a})/\lvert\mathrm{NPV}(\mathbf{a}^-)\rvert$	2.63
scheduling component			
eq. (19)	1.876		
BEAIRR			
$\mathrm{AIRR}(\mathbf{a}, \mathbf{V_0^e})$	14.64%		
capital component	4.723		
return component	0.397		
scheduling component			
eq. (22)	1.876		

Table 3a. NSE – cash flows and capital values

year	cash flows		capital values		
	a	**b**	$\mathbf{v^a}$	$\mathbf{v^b}$	$\mathbf{V_2^e}$
0	−4,272.26	−4,272.3	4,272.3	4,272.3	4,272.26
1	−25,700.88	−25,700.9	30,311.7	30,311.7	30,311.73
2	−14,106.40	−14,106.4	46,820.5	46,820.5	46,820.46
3	4,642.37	4,208.01	45,888.8	46,323.2	59,087.19
4	4,603.76	4,208.01	44,921.9	45,786.5	57,437.79
5	4,522.02	4,208.01	43,960.2	45,207.2	55,787.66
6	4,494.08	4,208.01	42,950.1	44,582.1	54,082.97
7	4,518.76	4,208.01	41,835.3	43,907.4	52,268.36
8	4,557.57	4,208.01	40,593.4	43,179.2	50,324.20
9	4,487.25	4,208.01	39,323.3	42,393.3	48,353.16
10	4,411.66	4,208.01	38,028.2	41,545.2	46,359.15
11	4,075.50	4,208.01	36,966.6	40,629.8	44,601.61
12	3,813.28	4,208.01	36,083.1	39,641.9	43,018.41
13	3,931.72	4,208.01	35,011.1	38,575.6	41,237.61
14	4,035.33	4,208.01	33,750.6	37,424.9	39,264.16
15	4,175.15	4,208.01	32,250.3	36,183.0	37,052.22
16	3,943.92	4,208.01	30,862.3	34,842.6	34,960.90
17	3,875.68	4,208.01	29,432.6	33,396.0	32,833.27
18	3,908.19	4,208.01	27,857.1	31,834.8	30,566.75
19	3,553.04	4,208.01	26,511.9	30,149.8	28,542.05
20	3,387.88	4,208.01	25,225.2	28,331.3	26,581.27
21	3,500.78	4,208.01	23,723.6	26,368.7	24,409.56
22	3,819.73	4,208.01	21,784.0	24,250.5	21,810.31
23	3,740.16	4,208.01	19,770.4	21,964.5	19,160.66
24	3,713.92	4,208.01	17,623.3	19,497.2	16,404.78
25	4,002.97	4,208.01	15,017.1	16,834.4	13,222.05
26	3,869.13	4,208.01	12,338.1	13,960.6	10,014.02
27	3,717.91	4,208.01	9,598.1	10,859.1	6,796.81
28	3,618.82	4,208.01	6,739.9	7,511.7	3,517.84
29	3,693.73	4,208.01	3,580.4	3,899.0	7,022.71
30	3,864.12	4,208.01	0.0	0.0	0.00
NPV	14,401.86	15,318.8	516,915.0	549,827.3	601,407.1

Table 3b. NSE – scheduling component

AIRR			
IRR (σ)	7.93%	$\lvert \mathrm{NPV}(\mathbf{a}^-) \rvert$	41,544.2
COC (r)	5%	$\mathrm{NPV}(\mathbf{b})$	15,318.8
$\mathrm{AIRR}(\mathbf{a}, \mathbf{v}^\mathbf{b})$	7.75%	interest component	0.369
scheduling component			
eq. (18)	0.94		
Average capital			
$\mathrm{NPV}(\mathbf{v}^\mathbf{a})/30$	17,230.50	Profitability index	
$\mathrm{NPV}(\mathbf{v}^\mathbf{b})/30$	18,327.58	$\mathrm{NPV}(\mathbf{a})/\lvert \mathrm{NPV}(\mathbf{a}^-) \rvert$	0.347
scheduling component			
eq. (19)	0.94		
BEAIRR			
$\mathrm{AIRR}(\mathbf{a}, \mathbf{V}_\mathbf{2}^\mathbf{e})$	7.51%		
capital component	1.094		
return component	0.860		
scheduling component			
eq. (22)	0.94		