

## **How the RCV Method Implies Excessive Returns on Capital Expenditure.**

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### **1. Introduction**

1.1 In February 2006, OFWAT and OFGEM issued a discussion paper dealing with certain aspects of the Regulatory Capital Value, (RCV), method of utility pricing: (ref: OFWAT/OFGEM 2006). This paper is a response to that discussion paper. One of the key issues highlighted in the discussion paper was the trend towards increased gearing in many utility companies. Rather than dealing with the specific items for discussion highlighted in the OFWAT/ OFGEM paper, this paper puts forward a radically different perspective on the reasons for the observed increase in gearing. The basic hypothesis advanced here is that an unwitting consequence of the RCV method is that it turns the undertaking of capital investment financed by fixed interest loans into a highly profitable activity for utilities- albeit, one that yields a concealed cash surplus, rather than a conventionally defined profit. The observed increase in gearing is therefore a rational action by the companies involved, designed to maximise the resulting financial surplus. Moreover, the financial surplus associated with the RCV method is likely to have significant distorting effects on a number of other aspects of the way utilities operate.

1.2 The nub of this paper is section 2, which develops financial modelling comparing the cash return to a utility resulting from a given level of capital investment under the RCV method, with the cash required to repay principal, and pay interest on the resulting debt, if the investment had been financed by fixed interest debt. The model developed depends on three parameters- namely, the length of asset life, the rate of inflation, and the interest rate. It is shown that the RCV method generates a substantial surplus of cash over that required to fund the base investment: to give an example, for an asset life of thirty years, inflation at 2.5%, and interest rate at 5%, the surplus generated by the RCV method would amount to 43% of the value of the capital investment. Investment itself is therefore potentially a hugely profitable activity for a utility.

Section 3 of the paper considers some of the likely implications. These include distortions of the gearing ratios of companies: likely distortions of their capital investment programmes: the likelihood of disproportionate returns on equity: and overcharging of consumers.

Section 4 considers whether there are other benefits of the RCV method, which could outweigh these drawbacks: the conclusion is that there are not.

Section 5 makes appropriate recommendations.

1.3 The background to this paper is work undertaken by Cuthbert and Cuthbert on various other aspects of water pricing in Scotland: (refs: Cuthbert and Cuthbert 2006). As the RCV method has now been implemented in Scotland, this has led to the more detailed consideration of the properties of the RCV approach which is reported on in this paper.

## 2. Modelling the Financial Implications of the RCV Method.

2.1 In this section, a simple model is set up to describe the investment activities of a utility operating in a form of steady state, where the utility is funding its investment through fixed interest debt. The objective of the modelling is to compare the revenue which would be raised from customers under the RCV method, with the revenue the utility actually requires to service its debt repayments and interest charges.

2.2 It is assumed that the utility starts out with a clean slate: and in every year from year 1 on carries out a fixed amount of real investment: (for simplicity, the annual amount of real investment is assumed to be 1). It is assumed that capital assets have a fixed life, of  $n$  years. It is assumed that the inflation rate each year is  $r$ , (expressed as a fraction): so the actual amount of investment in money terms from year 1 on is  $1, (1+r), (1+r)^2, \dots$  and so on. Finally, it is assumed that the utility finances its investment by borrowing at a fixed interest rate,  $i$ , (again, expressed as a fraction). There are therefore three parameters in the model, namely,  $n, r$  and  $i$ .

2.3 It is a standard result, (ref: Joskow, 2005, quoting Schmalensee), that, if the utility charges customers each year an amount equal to historic cost straight line depreciation of the capital assets, plus interest on outstanding debt, then this approach will satisfy the Net Present Value criterion for investment. In other words, this approach will generate sufficient revenue to repay the capital which has been borrowed, and give lenders a return on the loans equal to the opportunity cost of their capital. This approach is the so-called Brandeis formula, (Joskow, 2005), which is simply denoted here as the “historic cost” approach.

2.4 The historic cost approach, therefore, sets a useful baseline, which involves the full repayment of debt and interest charges from customer revenues. This is the baseline against which the revenues generated by the RCV approach will be compared. In terms of the model of a utility set out in paragraph 2.2, once the process has been running at least  $(n+1)$  years, the utility will enter a form of steady state, in which the real value of depreciation and interest charges each year under the historic cost method will be:-

$$\text{depreciation:-} \quad \frac{[1 - (1+r)^{-n}]}{nr}$$

$$\text{interest payment:-} \quad i \left[ 1 - \frac{1}{nr} (1 - (1+r)^{-n}) \right] / r \quad .$$

The utility will also maintain a constant real debt, given by

$$\text{real debt:-} \quad \left[ 1 - \frac{1}{nr} (1 - (1+r)^{-n}) \right] / r \quad .$$

Since a constant real debt implies a cash debt rising with inflation, this implies that the utility will have to borrow from the capital markets an amount of cash each year whose real equivalent is

$$\text{“real borrowing”:-} \quad \left[ 1 - \frac{1}{nr} (1 - (1+r)^{-n}) \right] \quad .$$

[Note that real borrowing, as defined in this sense as the real equivalent of the cash the industry will borrow each year, is exactly offset by the erosion of the industry’s

existing real debt by inflation. This explains the apparent paradox that that the industry has positive “real borrowing”, while its real debt remains constant.] The above formulae are derived in annex 1.

2.5 Now consider the following representation of the RCV method for setting charges. This is a slightly simplified representation of the method used by OFWAT in England and Wales, and now introduced in Scotland by the Water Industry Commission for Scotland.

Under the RCV method, the Regulatory Capital Value of the utility represents the current value of the capital assets employed: the value of the RCV is rolled on from year to year by uprating for inflation, adding on investment, and subtracting current cost depreciation. What is included in charges to customers are current cost depreciation, and a capital charge equal to the interest rate applied to the current RCV. Once the industry has reached a steady state, (after (n+1) years or more), the relevant real values are as follows.

$$\text{RCV:- } \frac{i(n+1)}{2} .$$

$$\text{depreciation:- } 1 .$$

$$\text{capital charge:- } \frac{i(n+1)}{2} .$$

[The main simplification in this representation of the RCV method is that this version of the RCV calculates the capital charge on the basis of the RCV uprated for inflation to the beginning of the relevant financial year, rather than the midpoint of the financial year, as applied in practice. Since depreciation and capital charges are regarded here as being paid at the end of the year, account should actually be taken of an additional year’s inflation in comparing RCV depreciation and capital charge with the historic cost figures derived in para 2.4. This is taken into account in the comparison in the following paragraph.

Another simplification is that the treatment of taxation is not considered yet.]

2.6 Given the formulae in paragraphs 2.4 and 2.5, therefore, it follows that, in the steady state, real charges to customers under the RCV method exceed what would be needed to repay debt and service interest charges under the historic cost method by the following expression:-

$$\left[1 + \frac{i(n+1)}{2}\right](1+r)^{-1} - \frac{[1-(1+r)^{-n}]}{nr} - i\left[1 - \frac{1}{nr}(1-(1+r)^{-n})\right] / r \quad (1)$$

(The  $(1+r)^{-1}$  factor is included in the first term in this formula to account for the difference in price base mentioned at the end of para 2.5: the effect of the inclusion of this term is conservative.)

Given that the historic cost method satisfies the NPV criterion, (i.e., repays debt and interest charges in full), the expression in formula (1) can be regarded as a measure of financial surplus under the RCV method. Before going on to look at the values of this

expression for some particular values of n, r and i, it is appropriate to make two comments:-

- a) Some element of surplus over and above the charges implied by the historic cost method is likely to be justifiable- e.g., to cope with uncertainty. So it would be inappropriate to jump to too facile a conclusion that all of the surplus is unjustified: this is a point which will be returned to later.
- b) Very little of the surplus given by formula (1) will actually appear in the form of observable profit in the current cost profit and loss account of the utility. This is because current cost depreciation, and interest charges on whatever debt the utility has, are allowable charges against profits. A more appropriate description of the quantity in formula (1) would be “concealed financial surplus”, rather than “profit”.

2.7 Table 1 illustrates the values of formula (1), for two selected interest rates, (5% and 7.5%), for asset lives of 10, 20, 30 and 40 years, and for a range of inflation assumptions. Note that the figures in the table represent the steady state real charge on consumers under the RCV method, less the charge which would have been required under the historic cost approach. Since the basic model assumes a steady state real annual investment of 1, the figures in the table can be interpreted as the concealed financial surplus expressed as a fraction of the level of investment.

**Table 1: Profit on RCV method above historic cost requirement, as fraction of capital investment.**

		Interest=5%			
		Asset life (years)			
Inflation (as fraction)		10	20	30	40
	0	0.000	0.000	0.000	0.000
	0.005	0.026	0.062	0.104	0.154
	0.01	0.051	0.119	0.198	0.288
	0.015	0.075	0.172	0.283	0.407
	0.02	0.097	0.221	0.360	0.511
	0.025	0.119	0.267	0.429	0.603
	0.03	0.140	0.310	0.492	0.685
	0.035	0.160	0.349	0.549	0.757
	0.04	0.179	0.386	0.601	0.821
	0.045	0.197	0.420	0.648	0.878
	0.05	0.214	0.452	0.690	0.929

**Interest=7.5%**

Inflation (as fraction)	Asset life (years)			
	10	20	30	40
0	0.000	0.000	0.000	0.000
0.005	0.028	0.070	0.122	0.185
0.01	0.055	0.135	0.233	0.348
0.015	0.080	0.195	0.333	0.492
0.02	0.105	0.251	0.423	0.618
0.025	0.128	0.303	0.505	0.731
0.03	0.151	0.351	0.580	0.830
0.035	0.172	0.396	0.647	0.919
0.04	0.193	0.438	0.709	0.998
0.045	0.213	0.477	0.765	1.068
0.05	0.231	0.514	0.816	1.131
0.055	0.249	0.548	0.862	1.187
0.06	0.267	0.580	0.905	1.238
0.065	0.283	0.609	0.944	1.283
0.07	0.299	0.637	0.979	1.324
0.075	0.314	0.663	1.012	1.360

It is immediately apparent that the financial surplus under the RCV method grows rapidly with each of  $n$ ,  $r$  and  $i$ . Furthermore, the figures in the table are large: to give an example, for  $n=30$ , interest = 5%, and inflation 2.5%, (corresponding, as regards the latter two parameters, to approximate current conditions in the UK), there is a financial surplus under the RCV method equivalent to 42.9% of the real level of investment. For interest = 7.5%, the surplus would be 50.5% of investment.

2.8 It is appropriate now to bring in the question of tax. If it is assumed that the utility has so managed its affairs that its debt is close to what debt would have been under the historic cost method, then the “historic cost” component of the RCV revenues will not be subject to tax: but the financial surplus given by formula (1) will be. Assuming that the utility pays 30% tax on this surplus still means that, in the  $n=30$ ,  $i=5\%$ ,  $r=2.5\%$  case, the utility would have a post tax surplus of  $42.9 \times 0.7 = 30.0\%$ . For interest = 7.5%, the surplus would be  $50.5 \times 0.7 = 35.4\%$ . Hence the post tax surpluses implied by formula (1) are still very substantial.

2.9 What the above means is that, under the RCV method, the mere act of undertaking capital investment funded by fixed interest borrowing yields a very substantial concealed financial surplus for the utility. As was noted in para 2.6a, some level of surplus is likely to be justifiable- e.g., to allow a margin for contingencies. However, the levels of surplus typically observed in table 1 are so large that they are likely to have a distorting effect on utility behaviour: this is a topic discussed in the next section.

2.10 As the final topic in this section, it is worth looking at another interesting implication of the historic cost model: namely, the question of what the gearing would be for a utility operating under the historic cost model, (that is, funding its capital expenditure from debt, and charging customers historic cost depreciation plus interest.) Even though such a utility is funded entirely by debt, its RCV will be greater than its debt, because in calculating the RCV, all elements of the capital stock are

valued at current prices: while past debt is fixed at historic prices. The steady state ratio of debt to RCV for such a utility is given by

$$\text{gearing ratio:- } \frac{2\left[1 - \frac{1}{nr}(1 - (1+r)^{-n})\right](1+r)}{r(n+1)},$$

and is a function only of r and n. Table 2 shows the values of this ratio, expressed as percentages, for a range of values of r and n.

**Table 2: Gearing under historic cost model**

Inflation (as fraction)	Asset life (years)			
	10	20	30	40
0	100	100	100	100
0.005	99	97	95	94
0.01	97	94	91	88
0.015	96	91	87	83
0.02	94	89	83	79
0.025	93	86	80	74
0.03	92	84	77	71
0.035	91	81	74	67
0.04	89	79	71	64
0.045	88	77	68	61
0.05	87	75	66	58
0.055	86	74	64	56
0.06	85	72	62	54
0.065	84	70	60	52
0.07	83	68	58	50
0.075	82	67	56	48

2.11 Table 2 shows how the gearing for an entirely debt funded utility might nevertheless be surprisingly low: for example, for inflation at 5%, and an asset life of 30 years, gearing would be 66%.

2.12 The most important implication, however, of table 2 and its associated theory, is what it says about the concept of gearing. It is implicit in much that is written about gearing that there are essentially two components of RCV, namely, debt and equity. In fact, as table 2 shows, gearing ratios can be substantially less than 100, even for an entirely debt funded utility, in which there is no equity at all. A proper decomposition of RCV into its contributory components would distinguish four different components: namely, debt funding, equity funding, funding from retained profits, and the effect of inflation in enhancing the value of capital assets. For the debt funded utility being considered here, only the first and last of these components contribute to RCV. Failure to separate out these components of RCV means that much conventional discussion of gearing ratios is likely to exaggerate the importance of the equity contribution to RCV. If the different contributory components to the funding of RCV are not properly distinguished, then it is likely to be impossible to work out a system for rationally apportioning the return on capital to the correct recipients.

### 3. Likely Implications for Utility Behaviour.

3.1 This section discusses the likely implications of the above theory for the behaviour of utilities. It is argued that the likely effect will be to materially distort a number of important aspects of behaviour.

### 3.2 Distortion of Gearing Ratios

Since capital investment financed by fixed interest debt yields a substantial concealed financial surplus, the effect is likely to be that utilities increase their gearing ratios to benefit from this. This could account for the observed increase in gearing for, for example, the water and sewage companies in England. Given the size of the concealed financial surplus, the normal risks associated with high gearing will be more apparent than real. Given this, owners of companies will have little incentive to inject equity capital, which would merely dilute the return on existing equity.

### 3.3 Distortion of Capital Programmes.

If capital investment in itself is a highly profitable activity because of the return it generates in charges on consumers, this may well distort the capital investment programme itself. For one thing, utilities may pay insufficient attention as to whether a given capital project is justified in terms of its physical return to the utility: so the utility may over-invest in intrinsically poor projects. Moreover, as can be seen from table 1, the financial surplus on a project increases with increasing length of asset life: this may encourage utilities to invest in long term projects disproportionately, at the expense of short term projects. In the extreme, since in the water industry infrastructure renewal projects are funded straight from revenue, and therefore generate no RCV surplus, this may help to explain the water companies' traditional relative unconcern about detecting and repairing leaks. In fact, if reducing leaks saved enough water to reduce the requirement for long term capital investment, this would be financially disadvantageous to the utility.

### 3.4 Danger of a Disproportionate Return on Equity.

As has been noted above, most of the financial surplus on investment is concealed, and will not show up directly as profit in the current cost accounts of the utility. It is not immediately clear, therefore, how this surplus can be easily removed in the shape of dividends for equity holders. However, in the case of the water industry in England, another element in the regulatory accounts becomes relevant at this point. This is the so-called "financing adjustment", which represents a notional income element in the regulatory current cost profit and loss account, representing the benefit received through the eroding effect of inflation on cash debt. It turns out that, if the debt of the company is approximately equal to the level of debt implied by the historic cost model, then the financing adjustment will typically be of the same order of magnitude as the concealed financial surplus accruing under the RCV method: (the approximation is very good for asset lives of around 10 to 15 years: for longer asset lives, the surplus will be greater than the financing adjustment.) Because of the existence of the financing adjustment, the effect is that equity holders can remove a large part of the financial surplus generated by the RCV method from the company, without pushing retained profits into the negative.

3.5 Is there any evidence of an excessive return being taken on equity? At this point, the discussion in para 2.12 above becomes relevant. A much better indicator of the true return on equity is to relate dividends to the actual amount of capital which

has been raised by the company by means of equity, rather than to the quantity (RCV-debt), (since, as has been noted in para 2.12, this latter quantity also includes components relating to capital financed from revenue, and the effect of inflation on RCV.) It is revealing to perform the relevant calculation for the water and sewerage companies in England, over the period since the mid 1990's, when the RCV method was introduced. As OFWAT has confirmed, the amount of capital raised through equity is given as the sum of the terms "called up share capital" and "share premium", in table 7 of (OFWAT, 2005), and corresponding tables in earlier volumes. Table 3 shows dividends expressed as a percentage of this amount:-

**Table 3.**  
Water and Sewerage Companies in England and Wales: Dividends as percentage of called up share capital plus share premium.

1996/97	22.2%
1997/98	34.5%
1998/99	32.4%
1999/2000	18.6%
2000/01	19.3%
2001/02	13.9%
2002/03	23.5%
2003/04	18.4%
2004/05	18.6%

The figures are striking, and suggest that the return to the equity capital actually raised by the water and sewerage companies indeed appears to be grossly excessive.

#### 3.4 Excessive Customer Charges.

The final implication, of course, is that, since the financial surplus generated by the RCV approach arises directly from charges on customers, customers will be being overcharged. Overcharging, however, will not just arise as a direct effect. As has been argued above, the RCV approach will result in significant sub-optimalities in investment decisions: the resulting inefficiencies will, in due course, lead to cost increases which will also be passed on to customers, leading to additional, indirect, increases in customer charges.

### **4. Do Other Benefits Justify the Application of the RCV Method?**

4.1 It has been argued in this paper that the concealed financial surplus on capital expenditure that the RCV method yields has severe distorting effects on a number of important aspects of utility operations. Before finalising on this conclusion, it is appropriate to ask whether there might not be other important advantages of the RCV method which outweigh the disadvantages.

4.2 Under the RCV method, one of the primary arguments that has been advanced for basing the calculation of the cost of capital on the whole of the RCV is that this will ensure efficient use of capital resources. Effectively, this is an opportunity cost argument: all capital assets have to earn a return at least equal to the opportunity cost of employing those assets elsewhere.

4.3 While this argument at first sight seems strong, it fails to take into account the particular circumstances facing a utility. If the entity which was being required to earn a return on capital was a body facing a fixed budget constraint, (like a government department), or a firm which was a price taker in the market, (as would typically be the case in a competitive market), then the opportunity cost argument would indeed apply: in these circumstances, the body concerned would have a real incentive to dispose of any capital asset which did not justify, in terms of a real return, the capital charge being levied on it. However, the situation facing a typical utility is precisely the opposite: under the RCV method, the capital charge levied on an asset will build into the set of allowable costs which are passed on to consumers: disposing of a capital asset will lead to a reduction in RCV, which will impact negatively on charges from the next periodic review on. There is thus very limited incentive to dispose of capital assets: but every incentive to build up the capital base to earn the concealed financial return on capital investment. Because of the price maker status of a typical monopoly utility, the opportunity cost argument in fact operates in reverse: an overall capital charge which is then passed on to consumers is an incentive towards inefficient capital investment, rather than efficient deployment of capital.

4.4 As regards depreciation, two arguments put forward for the use of current cost depreciation, (as in the RCV approach), as against historic cost depreciation are that the former

(a) smooths out the type of sudden increase in charges which can result under historic cost depreciation, when a long lived asset comes to the end of its life and is replaced.

(b) prevents the types of pricing anomalies between competing firms which can arise under historic cost depreciation if the age profiles of the capital stocks differ between the two firms.

Both these arguments have weight in the appropriate context - which is the context of relatively small undertakings operating in a competitive market. The force of these arguments is much less for a very large undertaking like a typical utility, where the size and diversity of its capital stock is likely in any event to smooth out most of the discontinuities arising under historic cost depreciation. Further, for water utilities, which are effectively local monopolies, the second argument does not really apply.

4.5 Overall, it appears that neither of the arguments considered here in favour of different aspects of the RCV approach constitutes a strong defence of the method in the face of the kind of anomalies identified earlier in this paper.

## **5. Recommendations.**

5.1 This paper has put forward an explanation for the observed increase in gearing for utilities operating under the RCV method, as being a rational response to the concealed financial surplus which utilities obtain from the very act of undertaking investment financed by debt. The implications of this go wider and deeper than the specific issues highlighted in the original OFWAT/OFGGEM discussion paper. The resulting implications fall into two groups.

5.2 Need for a better decomposition of the sources of funding for RCV

It has been shown, (see paragraph 2.12), that the traditional decomposition of RCV into components funded by debt and equity is inadequate. What is required is a means of decomposing the funding sources of RCV into at least four components: debt, equity, retained profits, and the effects of inflation. Of these four components, three relate to funding sources where the underlying agents could, in justice, require appropriate rewards. Providers of loan capital require interest: and providers of equity require an appropriate dividend reward: but retained profits are funded from customer charges - and there is a strong case for saying that customers should be directly rewarded, probably by lower charges, for their share of the return being generated by the RCV of the company. Unless a satisfactory methodology for clarifying the funding sources of RCV is developed, therefore, then it will not be possible to devise a fair method of appropriately rewarding the relevant funding sources. The present method disproportionately rewards equity.

### 5.3 Need to devise alternative approach to RCV method

The major requirement is to modify the RCV approach, or to devise a satisfactory alternative, which overcomes the distortions implicit in the RCV approach which have been identified in this paper. It would be too facile to say that all that should be done is to return to the historic cost approach. Some margin over historic cost pricing is likely to be necessary, at least initially, to cope with contingencies and uncertainty: and in the longer run, if a contingency fund were to be built up, then the interest generated on this fund might permit customer charges to be set at a level below that implied by pure historic cost depreciation and interest charges. Either way, the reduction in charges relative to the RCV method would, as the analysis in this paper indicates, be substantial. Equally important, however, would be the correction of the other distortions implicit under the RCV method.

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### **Annex1. Derivation of Formulae in Section 2**

Historic cost.

1. In the steady state, (that is, after at least  $n+1$  years), depreciation in year  $t$  in cash terms will be the sum of components from the current and preceding  $n$  years, as follows:-

$$\begin{aligned}
 \text{Depreciation in year } t &= \sum_{k=0}^{n-1} \frac{(1+r)^{t-1-k}}{n} \\
 &= \frac{(1+r)^{t-1}}{n} \sum_{k=0}^{n-1} (1+r)^{-k} \\
 &= \frac{(1+r)^{t-1} [1 - (1+r)^{-n}]}{n [1 - (1+r)^{-1}]} \\
 &= \frac{(1+r)^t [1 - (1+r)^{-n}]}{nr} .
 \end{aligned}$$

Since depreciation and interest are assumed paid at the end of the relevant year, the appropriate deflation factor to deflate this cash expression to real terms is  $(1+r)^t$ . Hence

$$\text{Real depreciation} = \frac{[1 - (1+r)^{-n}]}{nr} .$$

2. Calculating the historic cost interest payment involves evaluating an

expression of the form  $\sum_{k=1}^n kx^{k-1}$ . This expression has the following value:-

$$\sum_{k=1}^n kx^{k-1} = \frac{d}{dx} \left[ \sum_{k=0}^n x^k \right] = \frac{d}{dx} \left[ \frac{x^{n+1} - 1}{x - 1} \right] = \frac{(n+1)x^n}{(x-1)} - \frac{(x^{n+1} - 1)}{(x-1)^2} .$$

Interest in year  $t$  in cash terms will be the sum of components from the current and preceding  $n$  years, as follows:-

$$\begin{aligned}
 \text{Interest in year } t &= \sum_{k=0}^{n-1} \frac{(1+r)^{t-1-k} (n-k)i}{n} \\
 &= \frac{i(1+r)^{t-1}}{n} \sum_{k=0}^{n-1} (n-k)(1+r)^{-k} \\
 &= \frac{i(1+r)^{t-n-1}}{n} \sum_{k=0}^{n-1} (n-k)(1+r)^{n-k} \\
 &= \frac{i(1+r)^{t-n-1}}{n} \sum_{k=1}^n k(1+r)^k \\
 &= \frac{i(1+r)^{t-n}}{n} \sum_{k=1}^n k(1+r)^{k-1}
 \end{aligned}$$

which, using the above, =  $\frac{i(1+r)^{t-n}}{n} \left[ \frac{(n+1)(1+r)^n}{r} - \frac{((1+r)^{n+1} - 1)}{r^2} \right]$

$$\begin{aligned}
&= \frac{i(1+r)^t}{r} \left[ 1 + \frac{1}{n} - \frac{(1+r)}{nr} + \frac{(1+r)^{-n}}{nr} \right] \\
&= \frac{i(1+r)^t}{r} \left[ 1 - \frac{1}{nr} (1 - (1+r)^{-n}) \right].
\end{aligned}$$

Deflating as before, this gives the required value for real interest. The corresponding expressions for debt under the historic cost model follow immediately on omitting the term  $i$  in the above.

### RCV

3. Since depreciation is at current cost under the RCV model, the cash value of the depreciation charge each year must be equal to the cash value of investment, once the steady state is reached: so the real value of the RCV depreciation charge is 1.

In the steady state, the capital stock, in real terms, is:-

$$\text{Capital stock} = \sum_{k=1}^n \frac{k}{n} = \frac{(n+1)}{2},$$

so the real RCV capital charge is  $\frac{i(n+1)}{2}$ .